

Physik und Anwendungen von weicher Röntgenstrahlung I

(Physics and applications of soft X-rays I)

Sommersemester 2015

Veranstalter :

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Vorlesungstermine + Ort :

3 SWS (plus 1 SWS Laborbesichtigung) 6 ECTS Punkte

LMU Garching, Coulombwall 1

Mittwochs, 13:00 –16:00 Uhr, Seminarraum 224

(bis einschliesslich 3.6.) mündliche Abschlussprüfung

Vorlesungsinfos : www.xray.physik.uni-muenchen.de

Literatur:

Eberhard Spiller: Soft X-ray Optics / (SPIE Optical Engineering Press, Bellingham, Washington, ISBN 0-8184-1654-1)

A. Michette : X-ray Optics

D.T. Attwood: Soft X-ray and Extreme Ultraviolet Radiation (Cambridge Univ. Press, ISBN 0-521-65214-6)

Sowie aktuelle Wissenschaftspublikationen...

Begleitendes Seminar :

Studentische Vorträge zu aktuellen Forschungsarbeiten
(z.B. Röntgenmikroskopie, EUV Lithographie, Röntgenlaser, ...)

Ort und Zeit : Mittwochs 16 c.t – 18 Uhr, Seminarraum 219

Outline :

•Basics :

- Introduction to physics in the soft X-ray range
- Basic processes, emission and absorption
- Maxwell-equations, wave equation
- Scattering by free and bound electron
- Scattering by multiple electron atom, atomic scattering factor
- Complex refraction index, wave propagation in media

•Thin films optics :

- Interfaces, Fresnel-equations, total external reflection, Brewster angle
- Multilayer-Optics, basics, theory and technology

•Sources :

- Introduction to Synchrotron Radiation, Wigglers and Undulators
- Basics of the Free Electron Laser (FEL)
- Laser Plasma (LIP) and discharge plasma (DPP) sources
- X-ray lasers, High Harmonics

•Nanooptics and diffractive optics

- Diffractive optics, amplitude and phase gratings
- Zoneplates and refractive optics
- Waveguides and capillary optics

•Detectors for soft X-rays

- CCD, photodiode, DLD detector

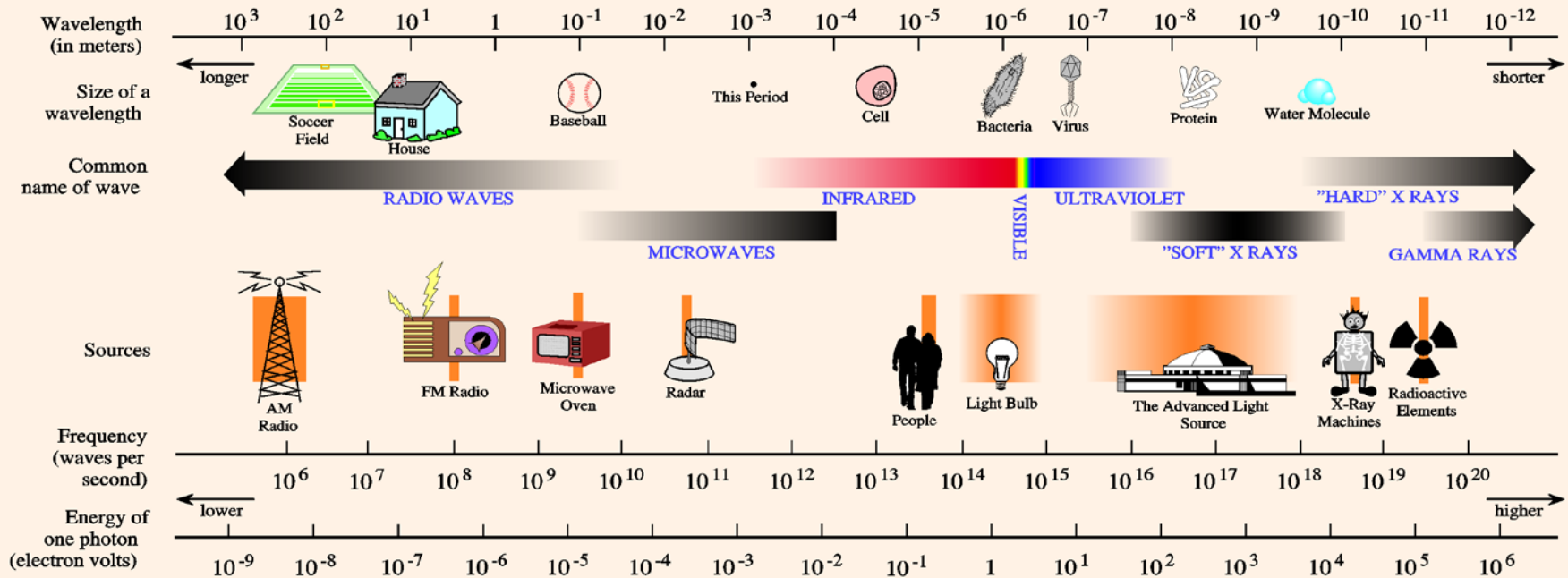
Technology and applications :

- **Soft X-ray microscopy and micro-spectroscopy**
- **Diffraction Imaging and Holography**
- **Extreme Ultraviolet Lithography**
- **Attosecond electron spectroscopy and microscopy**
- **Solar astronomy**

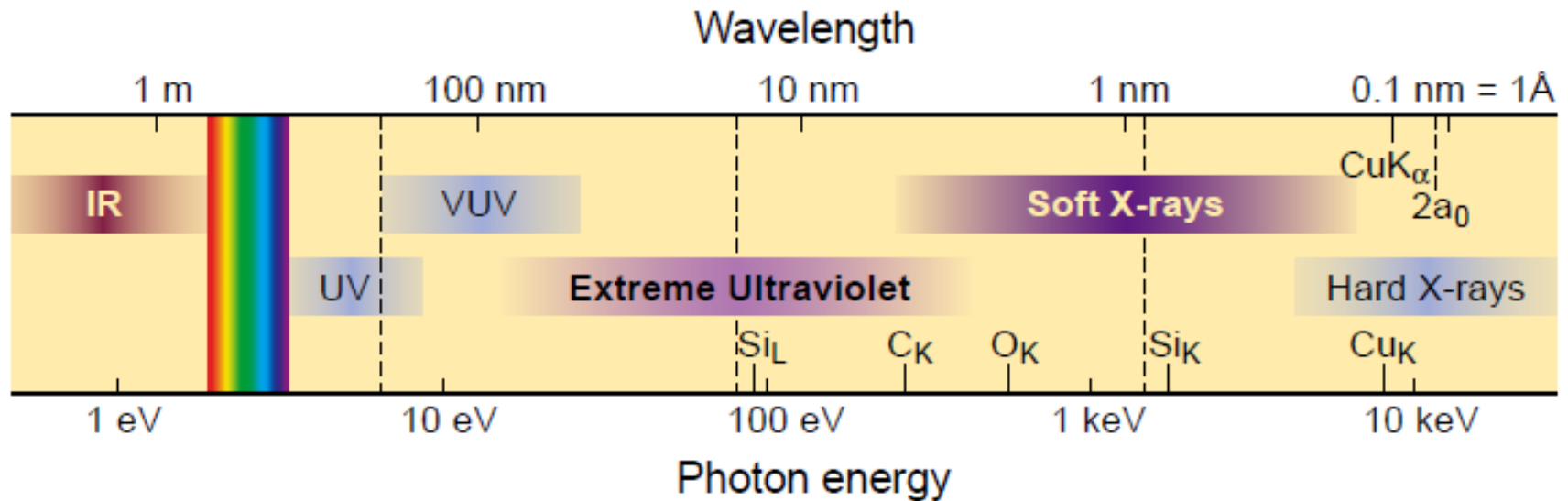
-

From radio waves to gamma rays

THE ELECTROMAGNETIC SPECTRUM



What is specific about the soft X-ray range ?



Short wavelength ~ 1 nm > see smaller features
> write smaller features

Core level electron energies > element specificity
> chemical specificity

Single cycle period sub-fsec > shortest electromagnetic pulses
> attosecond physics of e-dynamics

Resolution

How to determine the resolution of an optimal instrument / microscope ?

- Rayleigh-criterion:

$$d = \frac{0,61 * \lambda}{NA}$$

with λ = wavelength

NA = numerical Aperture

NA Animation : [NA](#)

<http://micro.magnet.fsu.edu/primer/java/nuaperture/index.html>

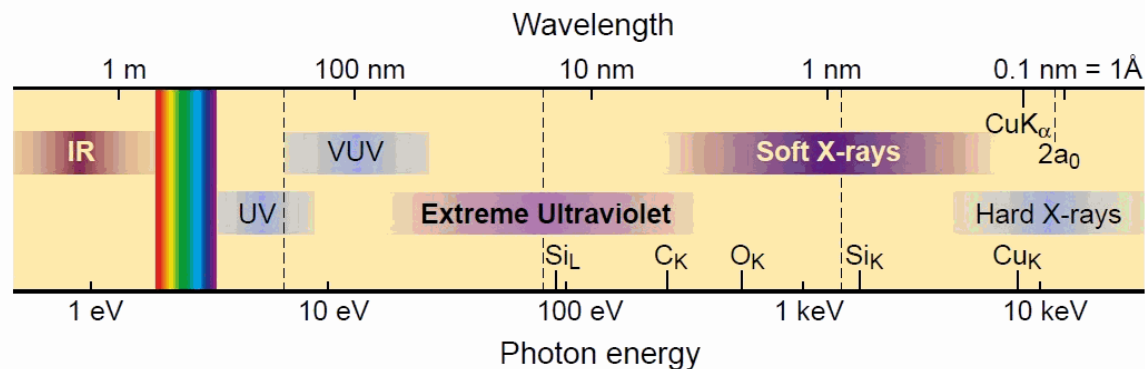
NA Immersion : [NA Immersion](#)

<http://micro.magnet.fsu.edu/primer/java/microscopy/immersion/index.html>

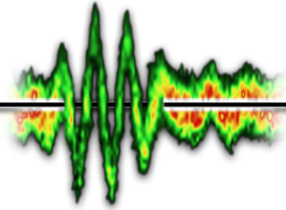
Rayleigh: [Resolution](#)

<http://micro.magnet.fsu.edu/primer/java/imageformation/rayleighdisks/index.html>

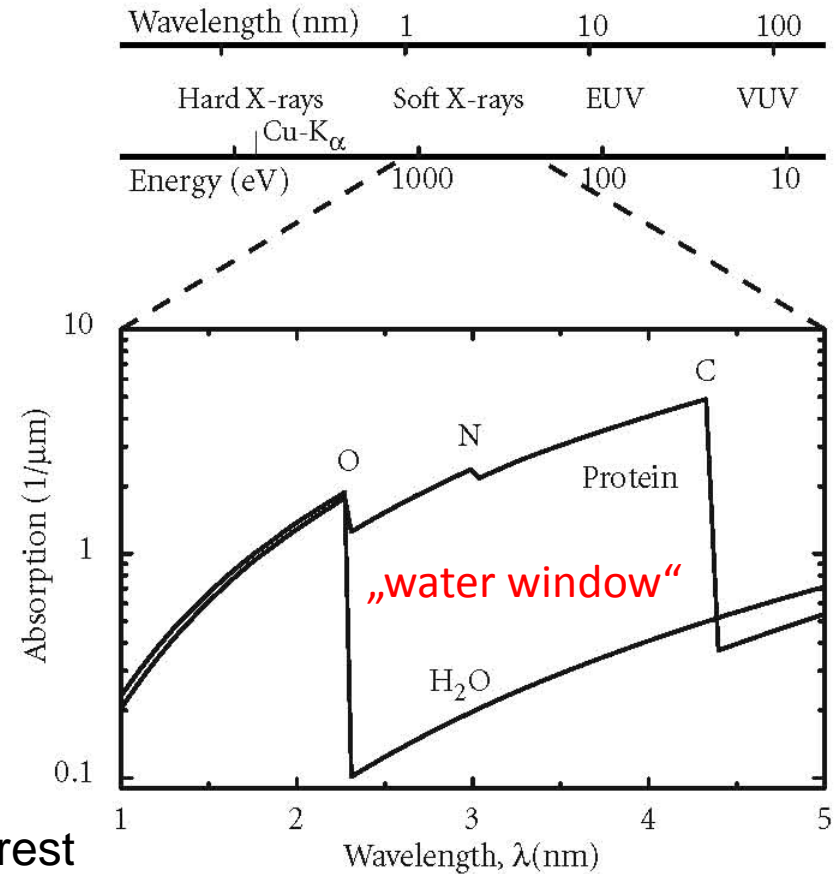
See smaller features with smaller wavelength !

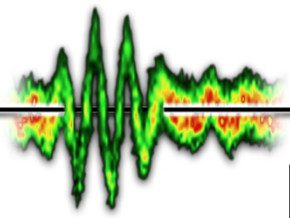


X-ray microscopy in the water window

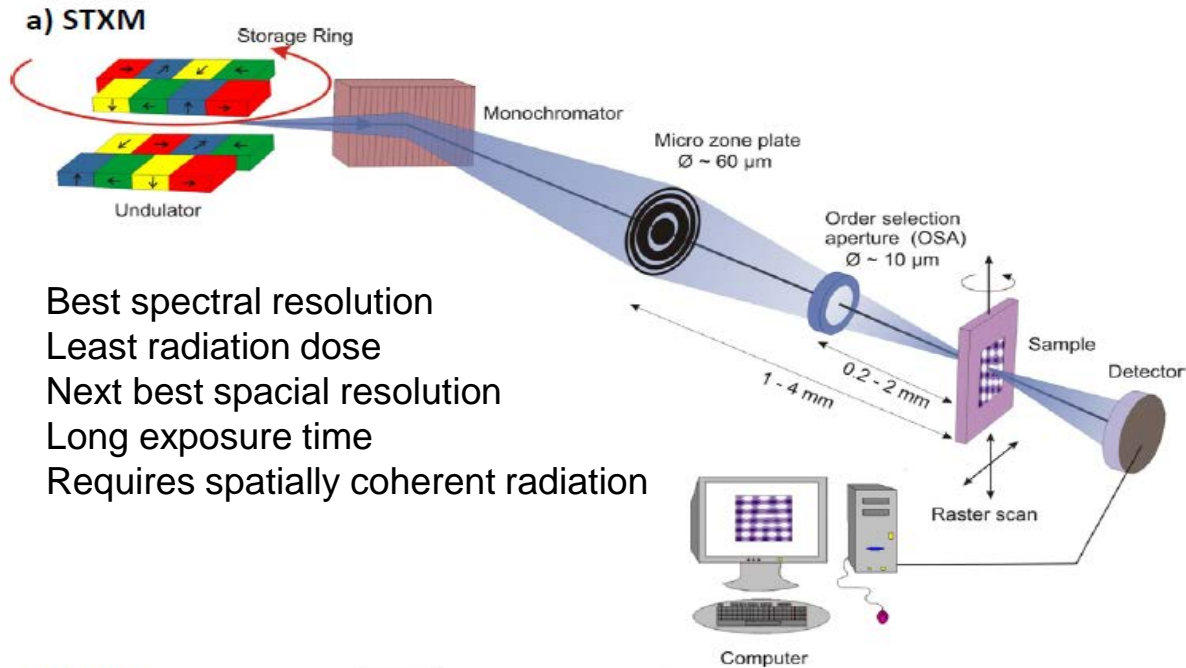


- Between the K-edge of carbon and oxygen
K-absorption edge O₂ : 2.28 nm = 543.1 eV
K-absorption edge C : 4.36 nm = 284.2 eV
- Natural contrast between materials containing carbon and water
- In-Vivo image of biological objects with high resolution
- Of high microscopic and spectroscopic interest

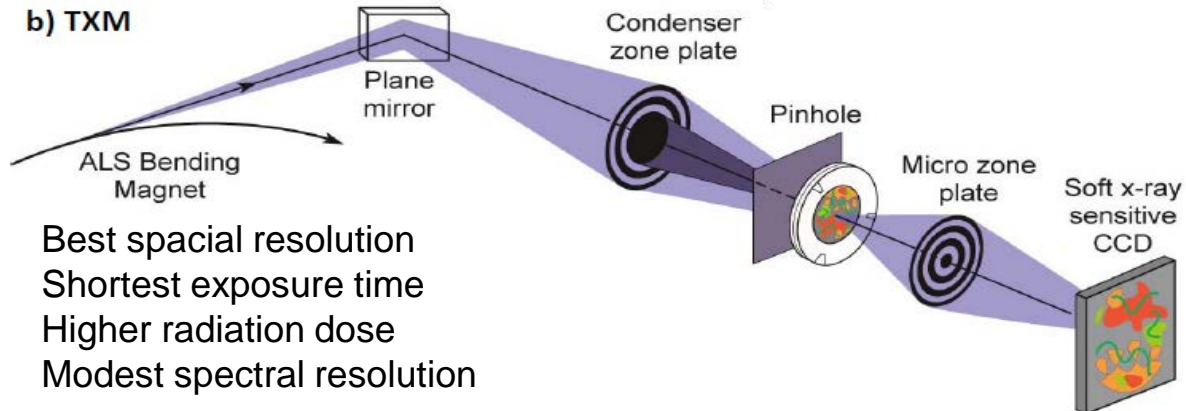




X-ray microscopy methods

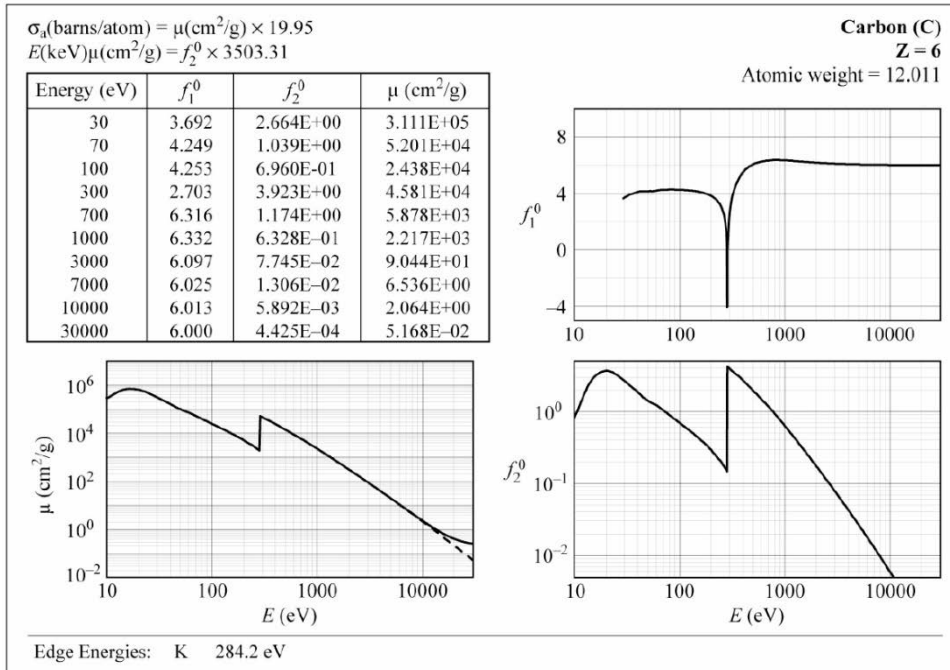
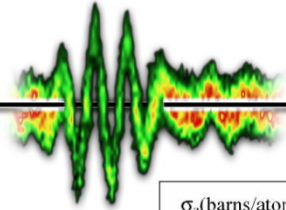


Best spectral resolution
Least radiation dose
Next best spacial resolution
Long exposure time
Requires spatially coherent radiation



Best spacial resolution
Shortest exposure time
Higher radiation dose
Modest spectral resolution

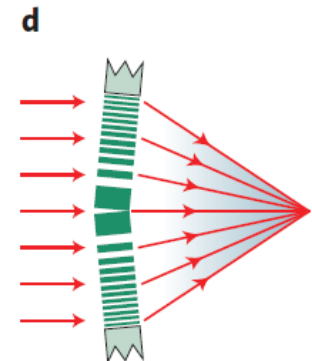
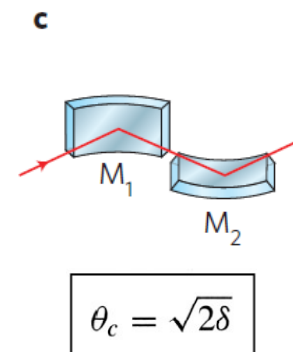
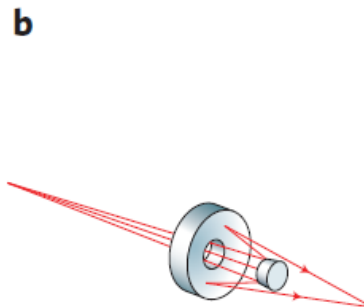
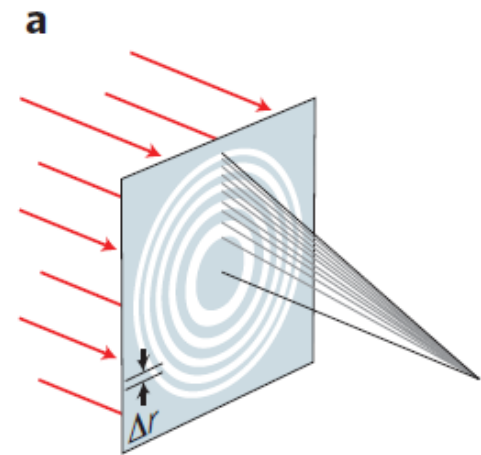
Refractive index, X-ray optics



(Henke and Gullikson; www-cxro.LBL.gov)

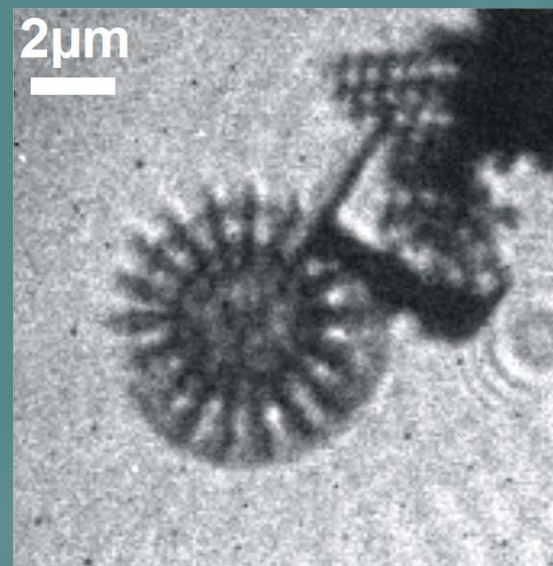
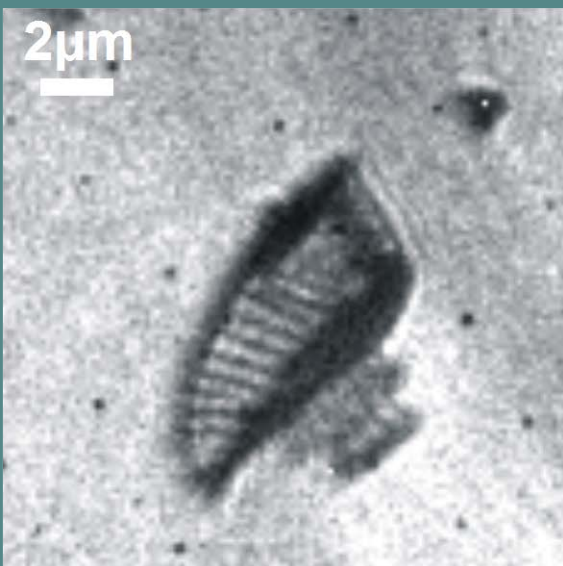
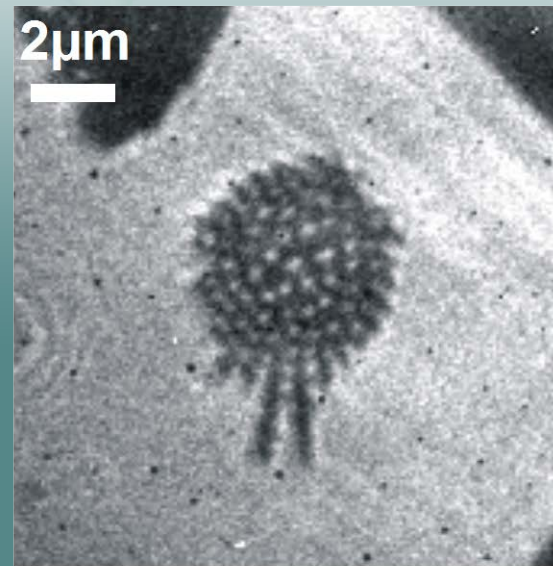
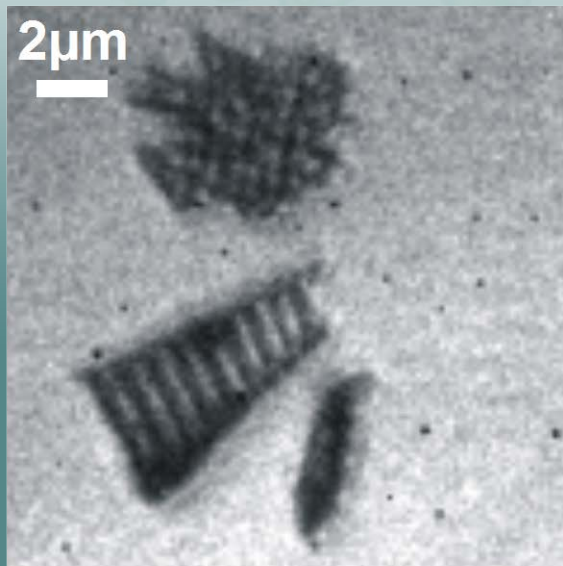
Ch02_F13VG.ai

Four common X-ray microscopy optics:



- a) Zone plate
- b) Schwarzschild optics
- c) Kirkpatrick-Baez mirror pair
- d) Laue lens

See smaller features : soft X-ray microscopy



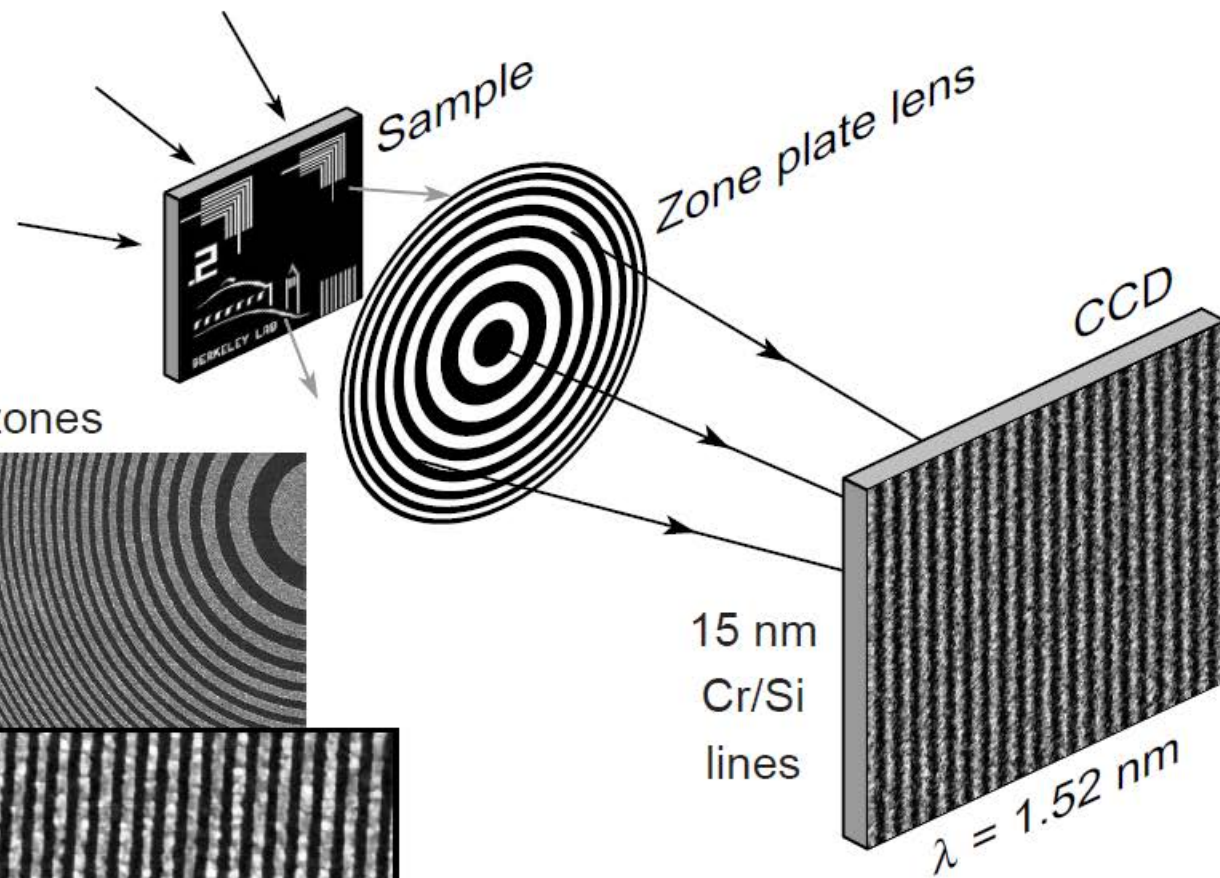
Soft X-ray microscopy
on diatoms (silica algae)

Eph = 97 eV

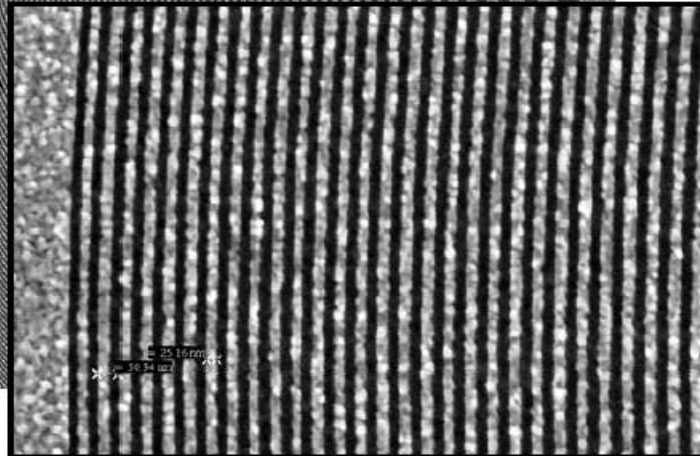
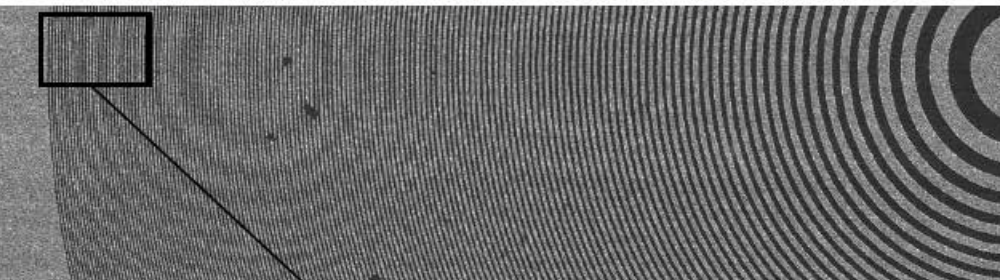
Lateral resolution :
< 200 nm



Soft X-Ray Microscopy at the ALS

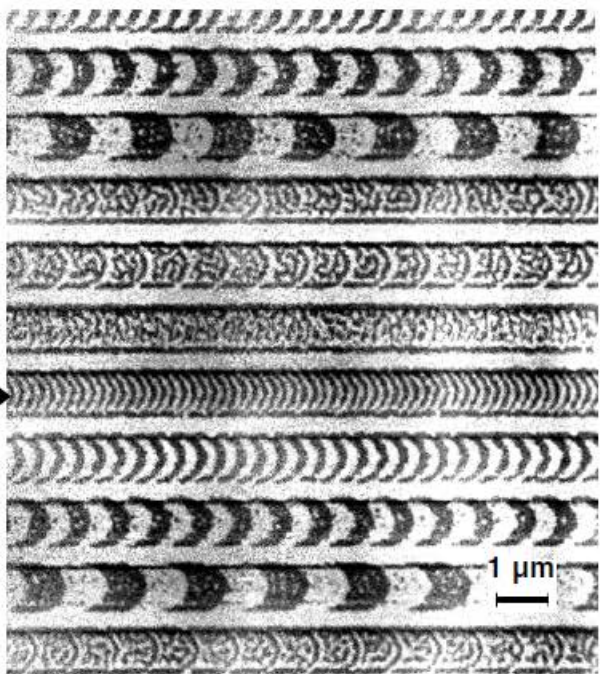


$\Delta r = 25 \text{ nm}$, $D = 63 \mu\text{m}$, $N = 618 \text{ zones}$



Courtesy of E. Anderson and W. Chao (CXRO/LBNL)

Magnetic Recording Materials

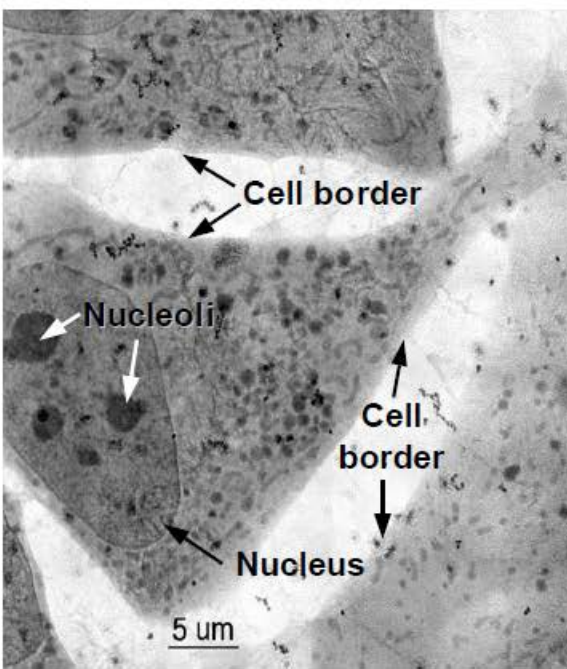


Fe L₃ @ 707.5 eV

FeTbCo Multilayer
with AL Capping Layer

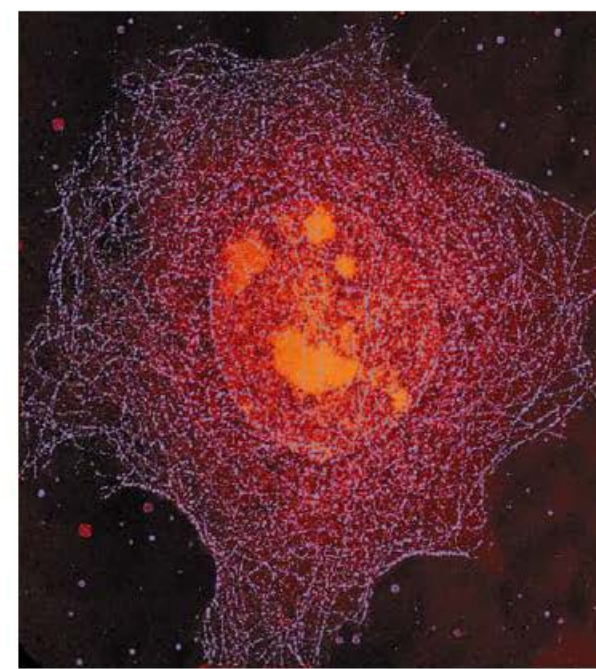
Courtesy of P. Fischer, Wuerzberg
and G. Denbeaux, CXRO/LBNL

Cryo Microscopy for the Life Sciences



Cryo X-Ray Microscopy
of 3T3 Fibroblast Cells

Courtesy of C. Larabell, UCSF
and W. Meyer-Ilse, CXRO/LBNL

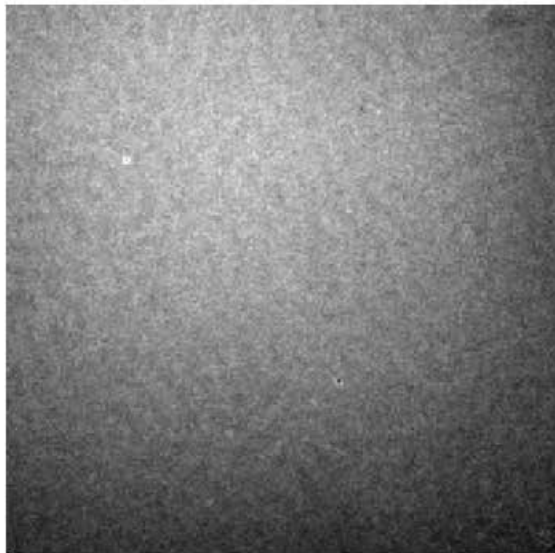


Protein Labeled
Microtubule Network

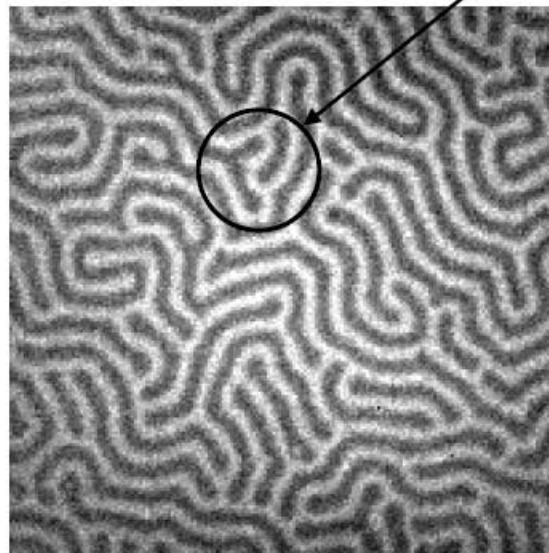


FeGd Multilayer

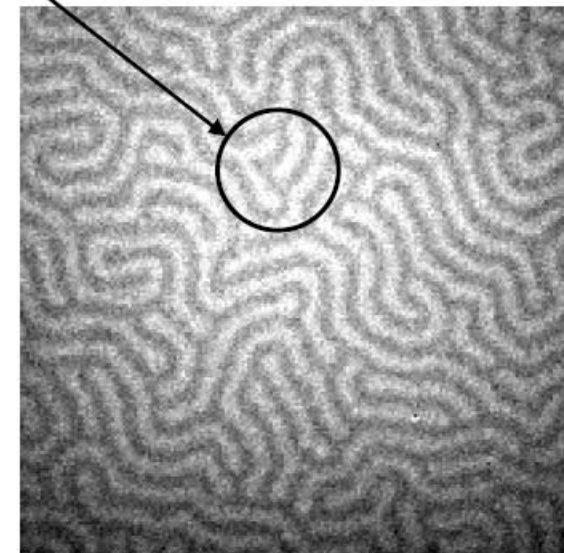
1 μm



$\hbar\omega = 704 \text{ eV}$
below Fe L-edges

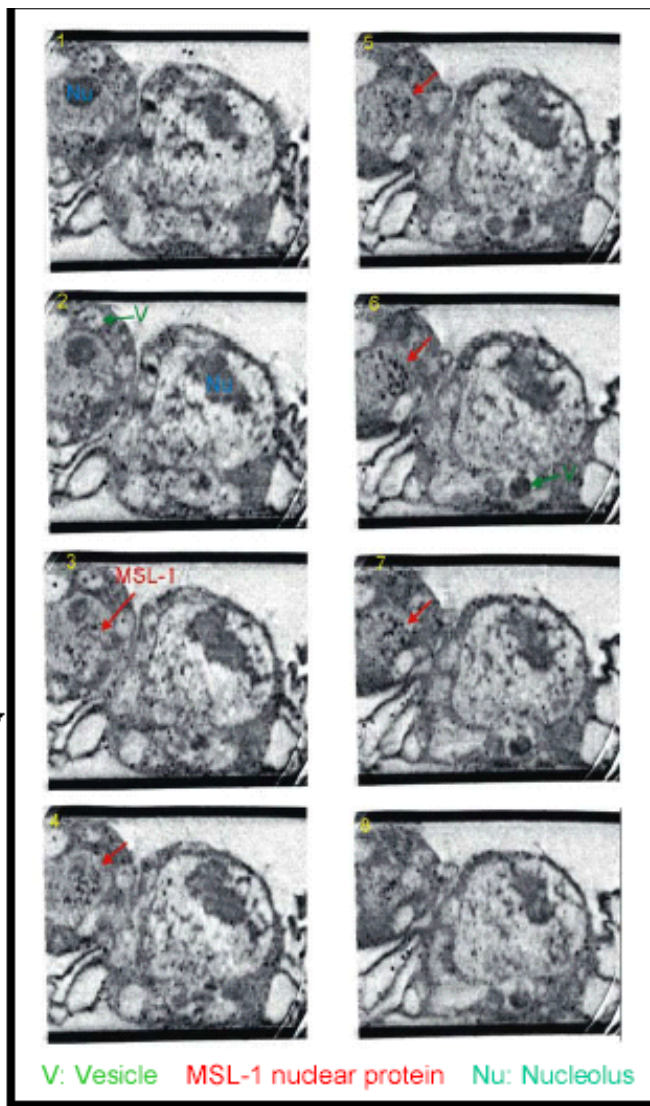
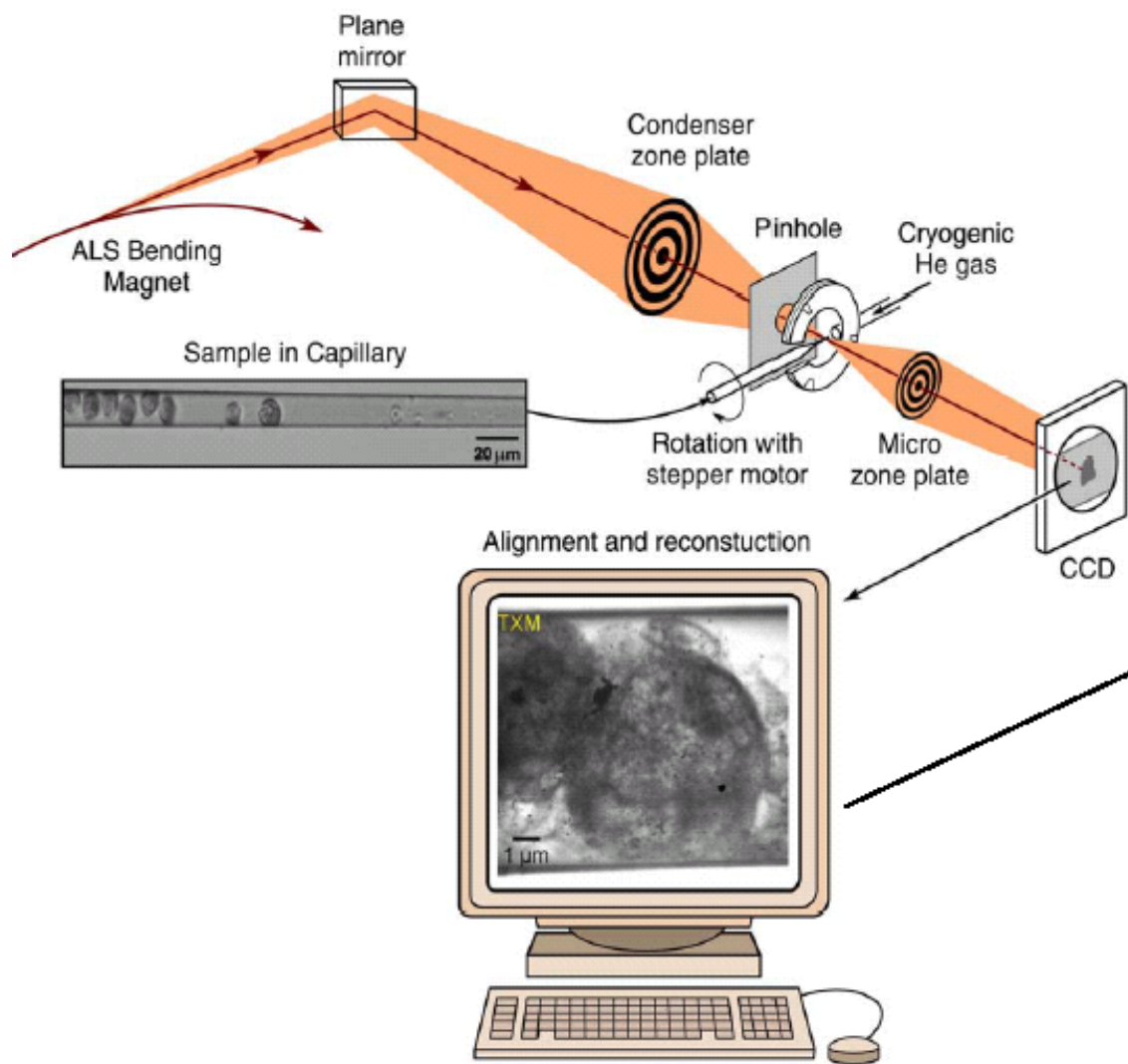


$\hbar\omega = 707.5 \text{ eV}$
Fe L₃-edge



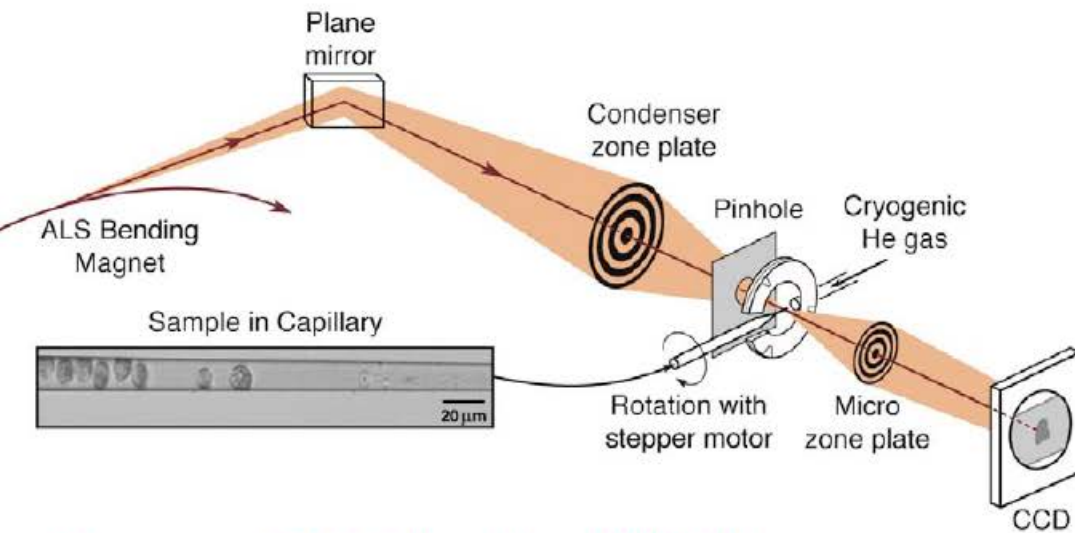
$\hbar\omega = 720.5 \text{ eV}$
Fe L₂-edge

Contrast reversal



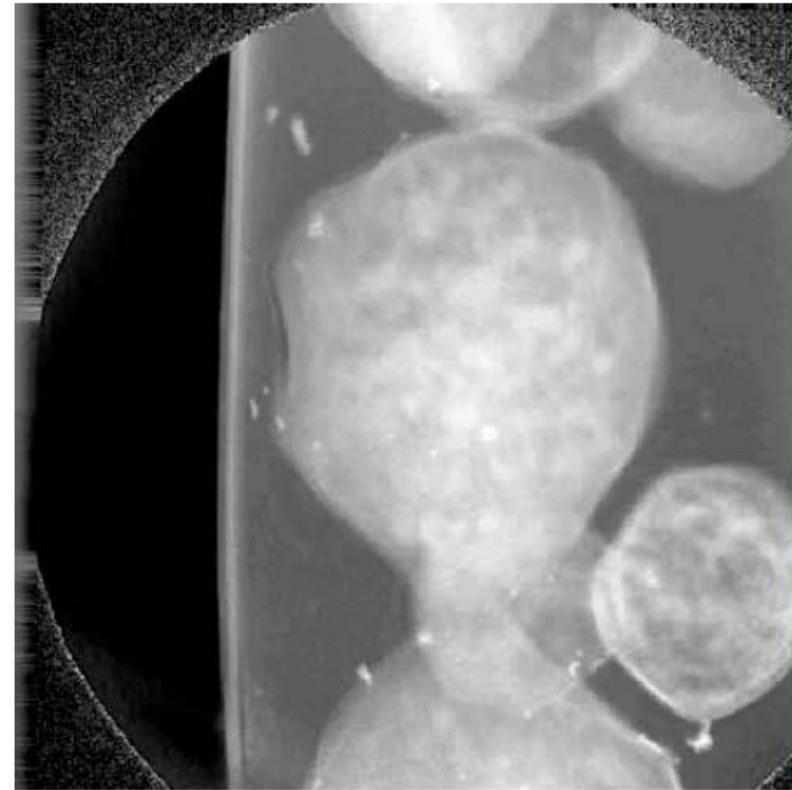
150 nm thick slices through the volume

Nanotomography of Cryogenic Fixed Cells



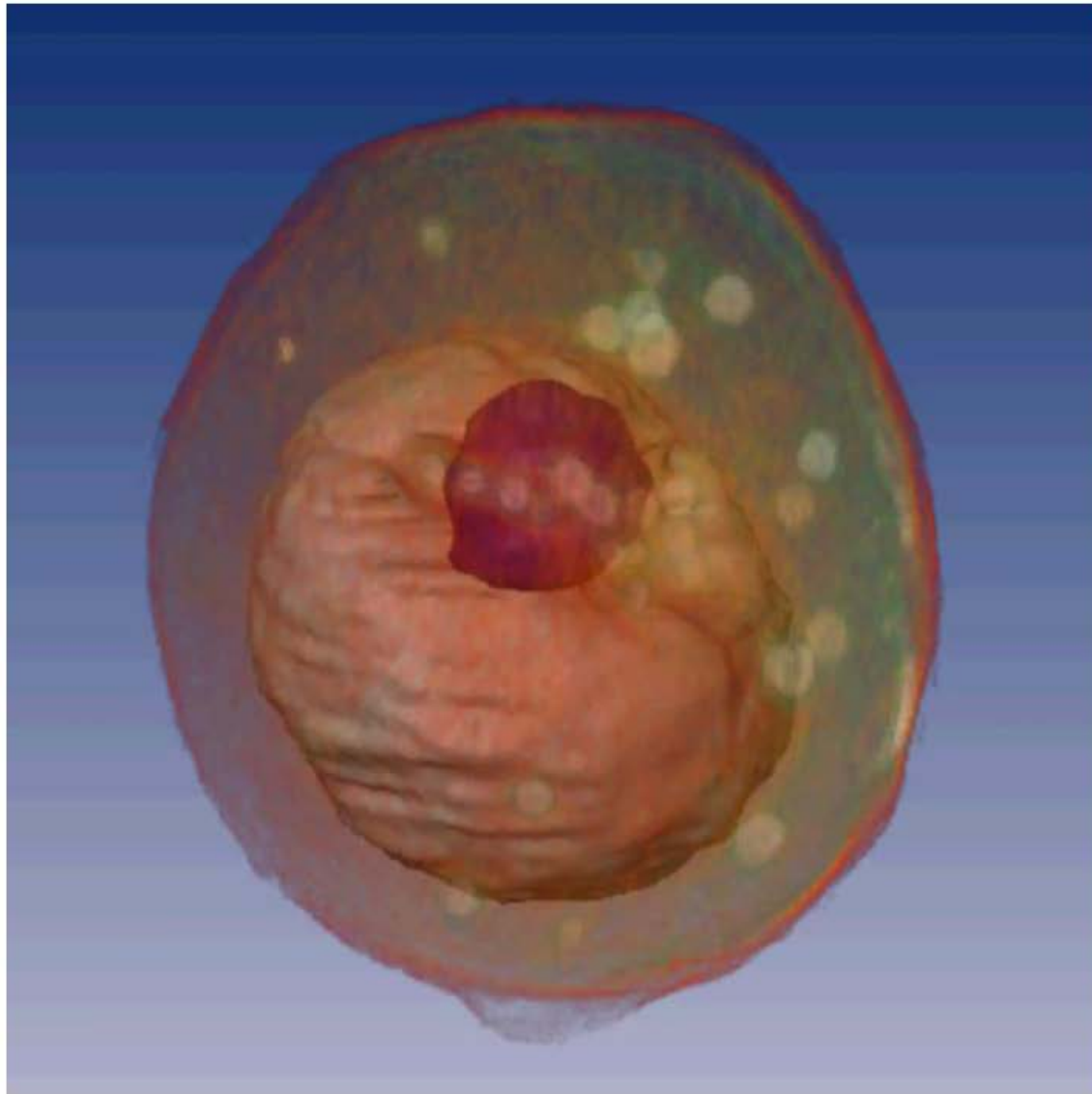
Courtesy of G. Schneider (BESSY)
Surf. Rev. Lett. 9, 177 (2002)

Soft X-Ray Nanotomography of a Yeast Cell



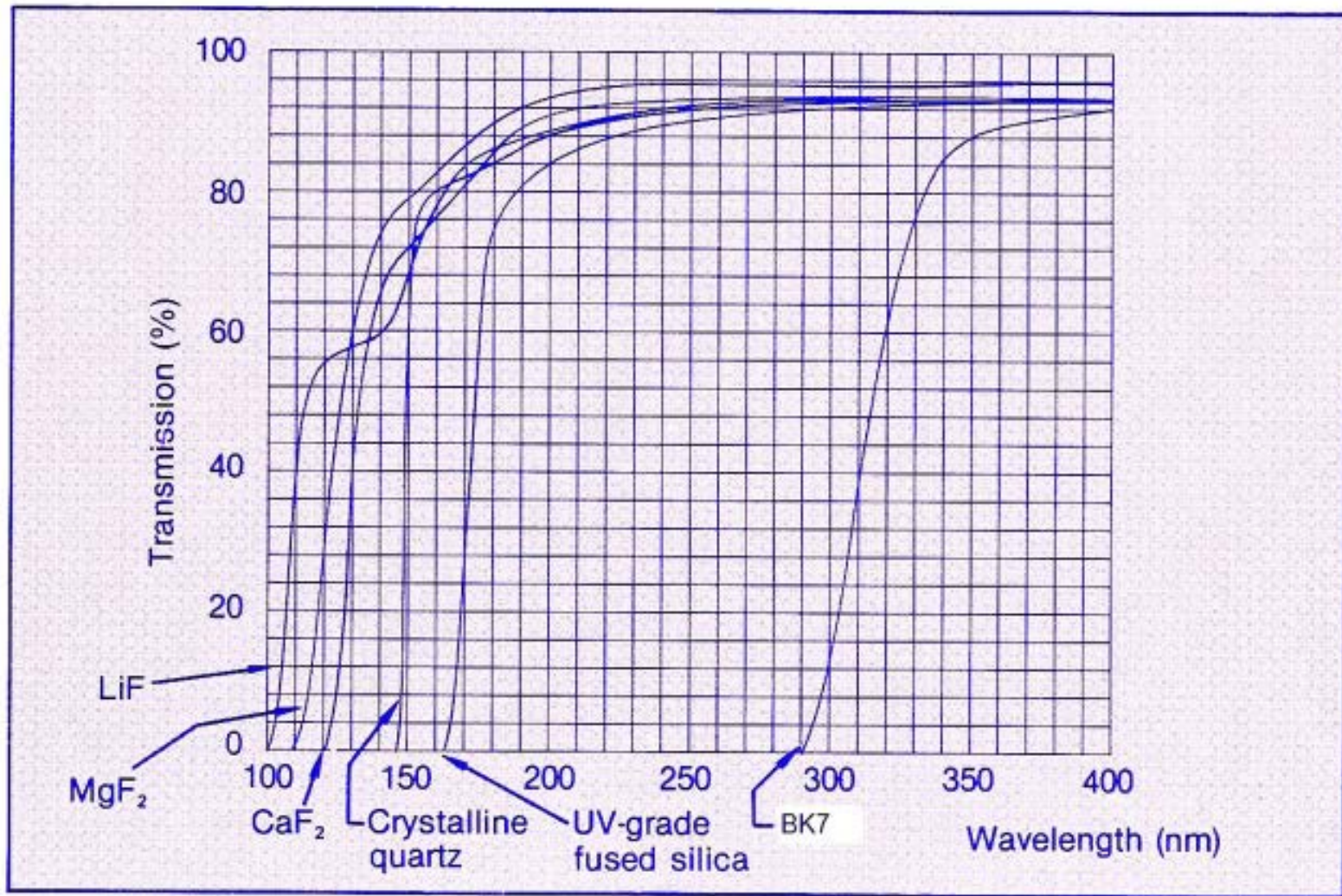
$\lambda = 2.5 \text{ nm}$

Courtesy of C. Larabell (UCSF & LBNL)
and M. LeGros (LBNL)



Courtesy of C. Larabell / UCSF & LBNL, and M. LeGros / LBNL

Transmission of fluoride materials in the DUV



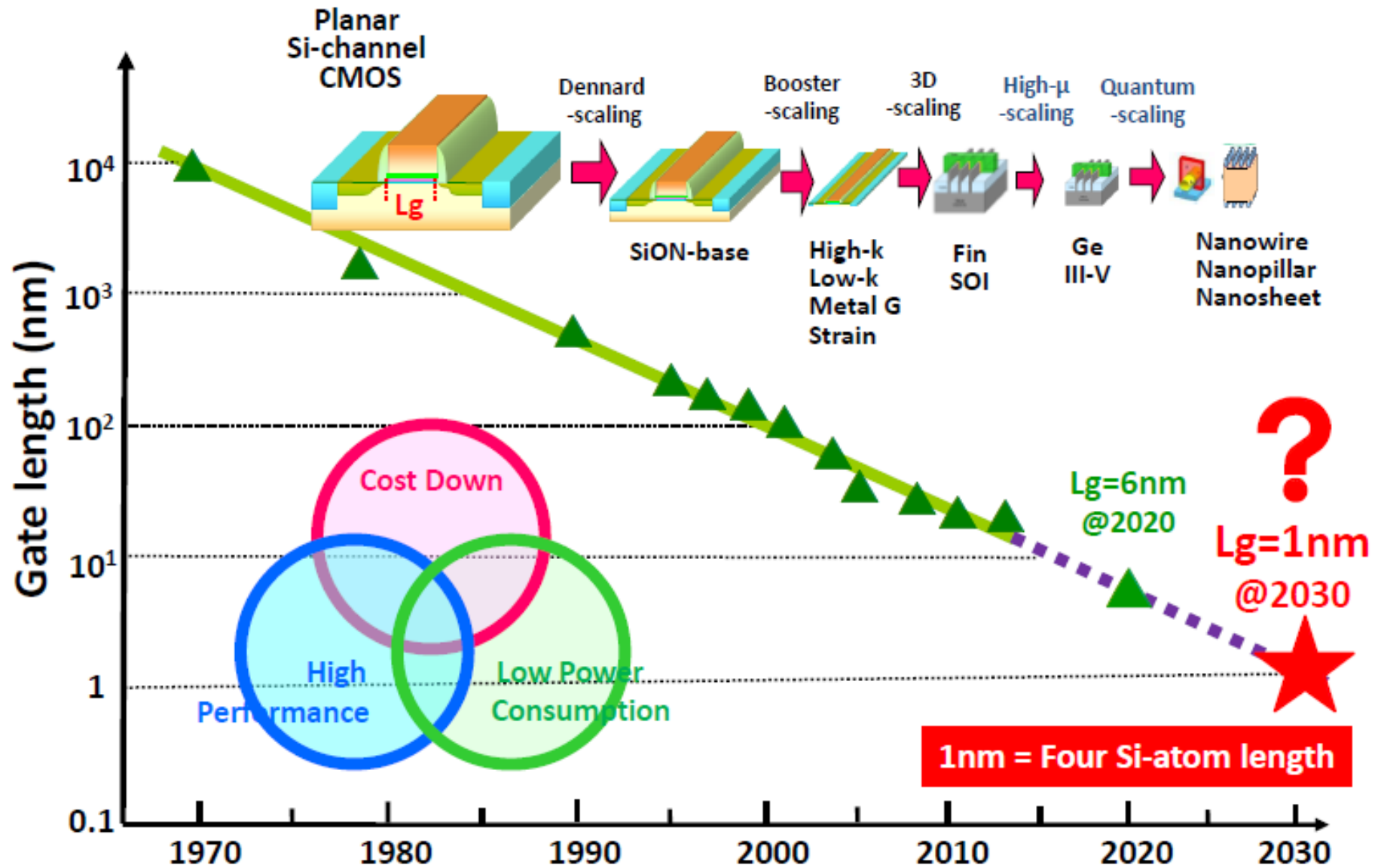
- No window materials below 110 nm wavelength
- No conventional transmission lenses possible !

193 nm DUV lithography objective (Zeiss)



> CaF aspherical lense optics

Write smaller features : Extreme Ultraviolet Lithography



Challenges of Extreme Ultraviolet Lithography

Source

- ✓ High power: 250W @IF
- ✓ Stability
- ✓ Long Life of Collector Mirror

Scanner

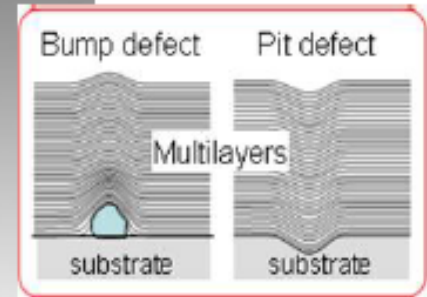
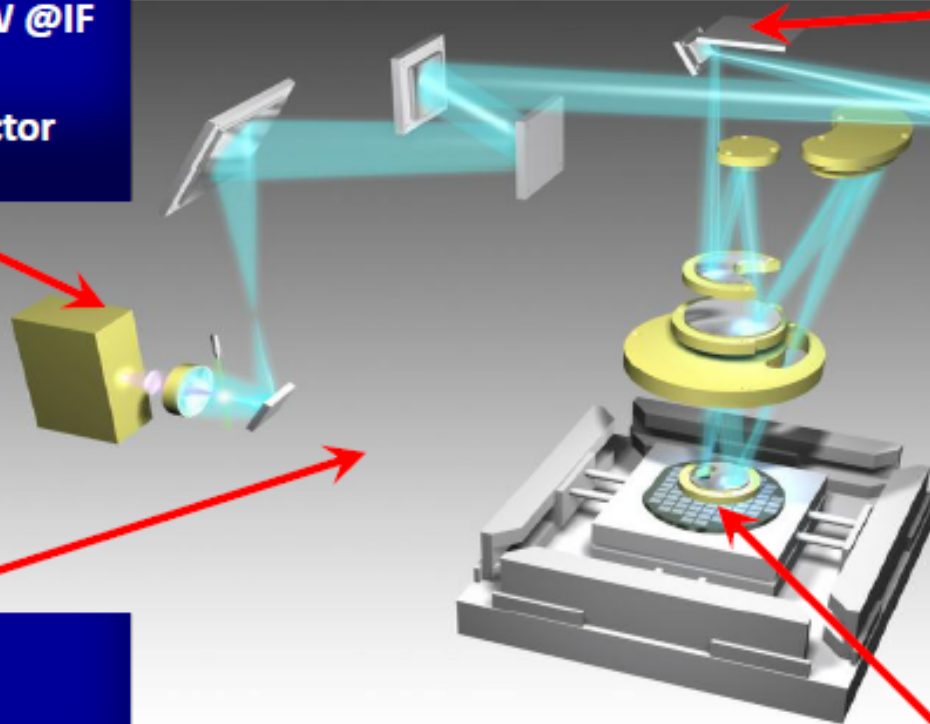
- ✓ Field data
- ✓ Higher quality / Long lifetime of optical components

Mask

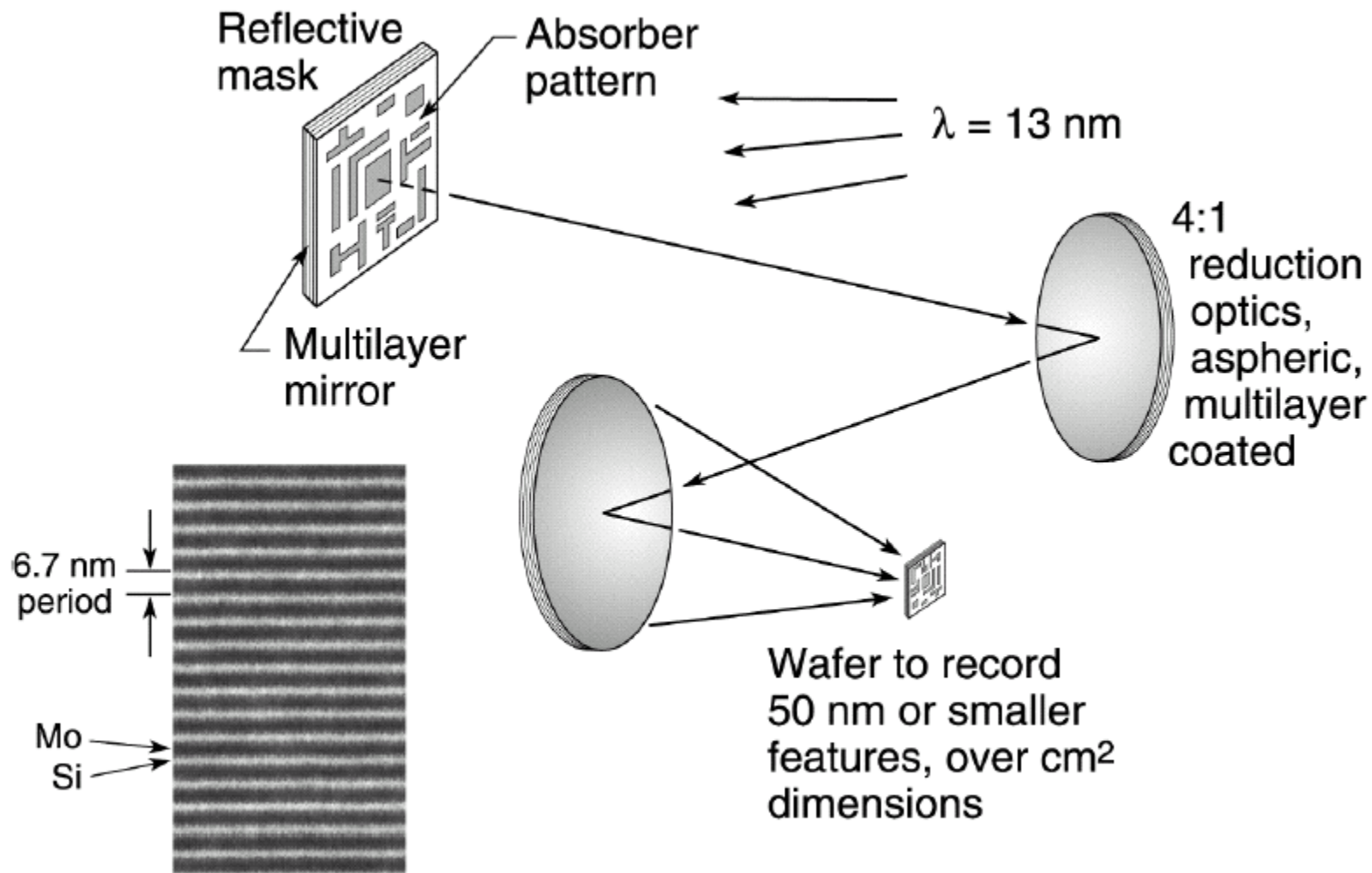
- ✓ Blank Inspection
- ✓ Patterned Mask Inspection
- ✓ Defect Review System
- ✓ Particle Free Handling

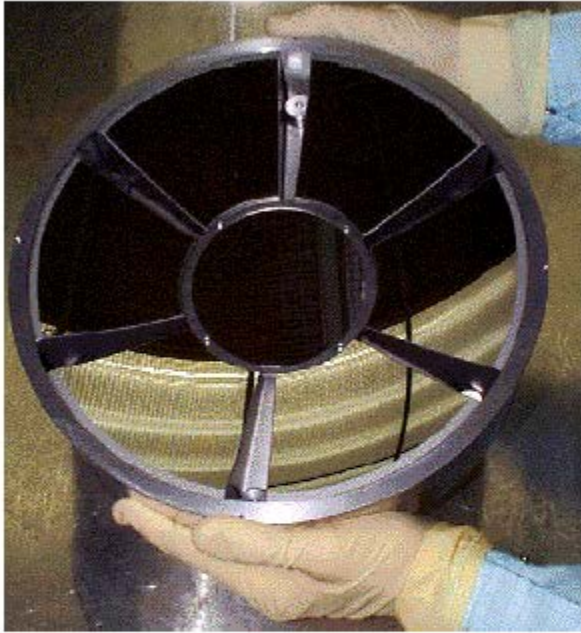
Resist

- ✓ Resolution < 20nmHP
- ✓ Sensitivity < 10mJ/cm²
- ✓ LER < 2nm
- ✓ Lower outgassing



Courtesy by EUVA



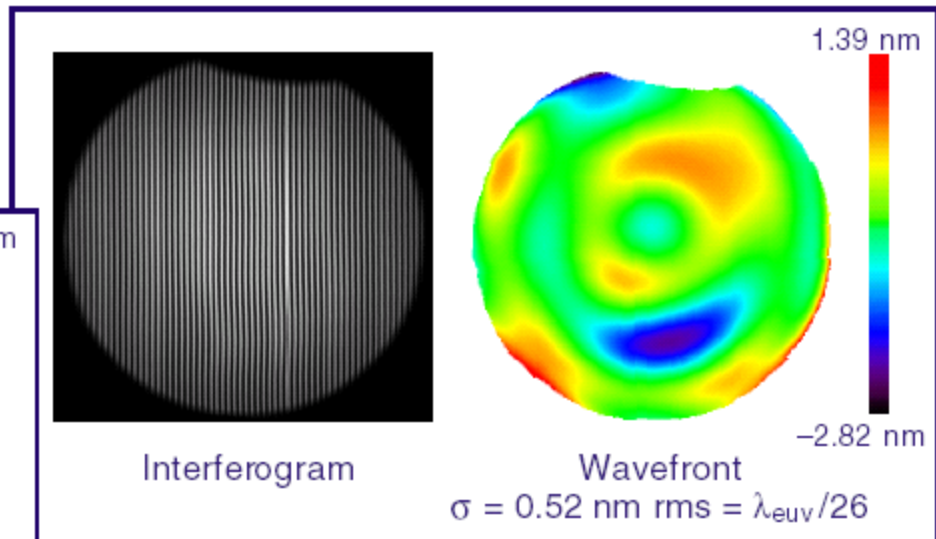
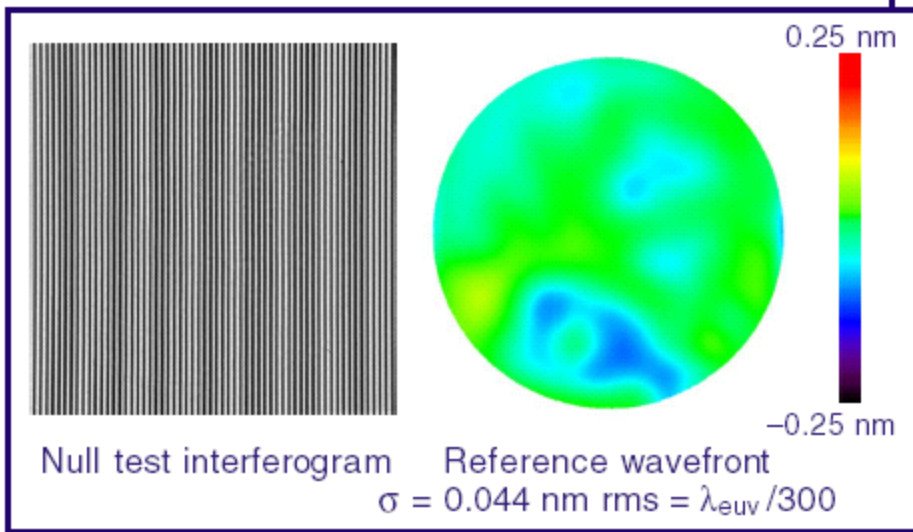
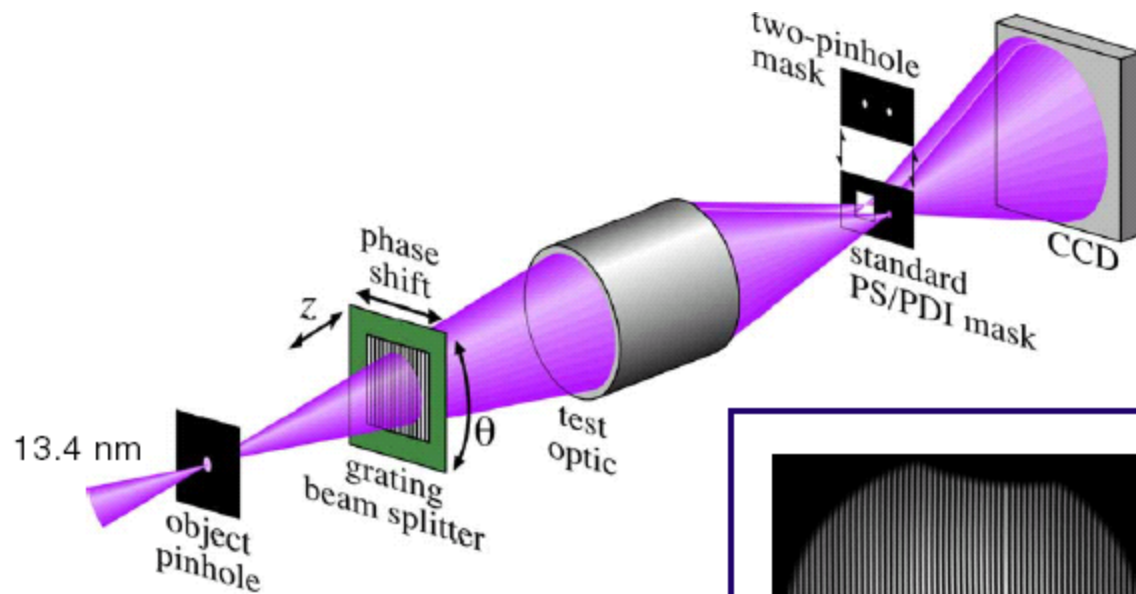


Condenser optic



Projection optic

Courtesy of J. Taylor and D. Sweeney / LLNL

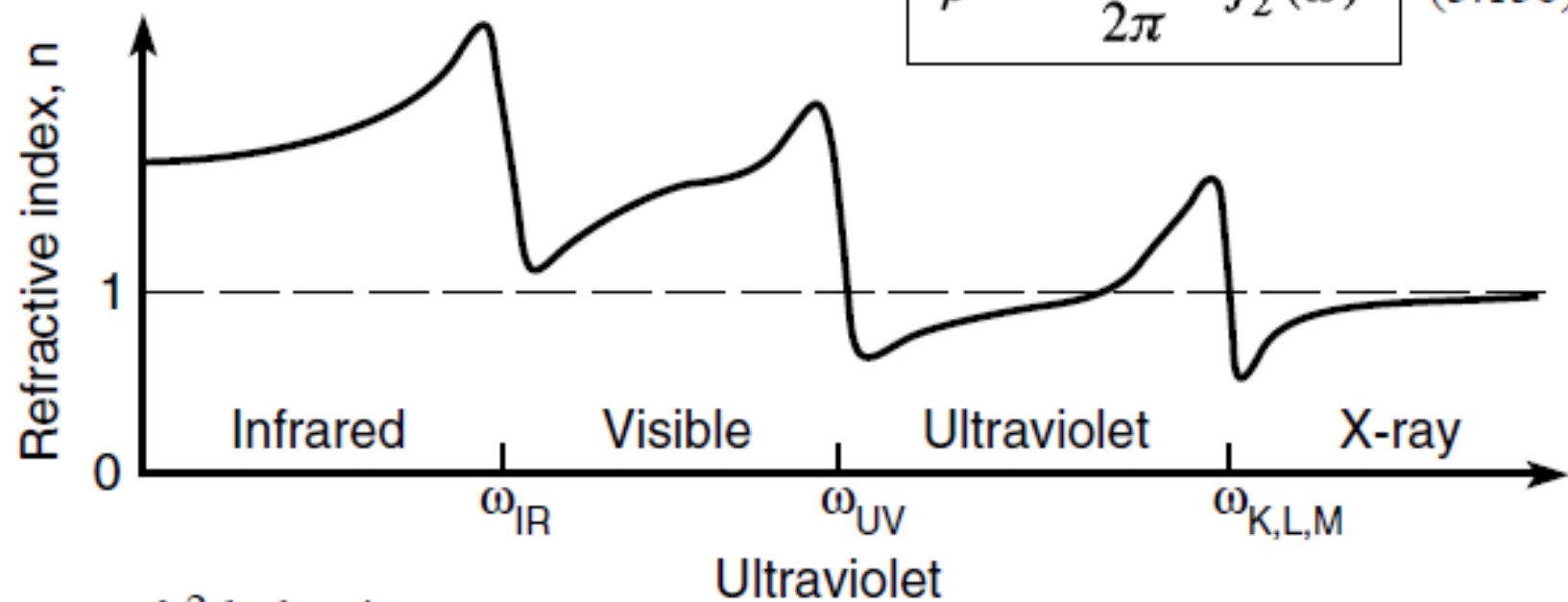


Complex index of refraction/atomic scattering factors

$$n(\omega) = 1 - \delta + i\beta \quad (3.12)$$

$$\delta = \frac{n_a r_e \lambda^2}{2\pi} f_1^0(\omega) \quad (3.13a)$$

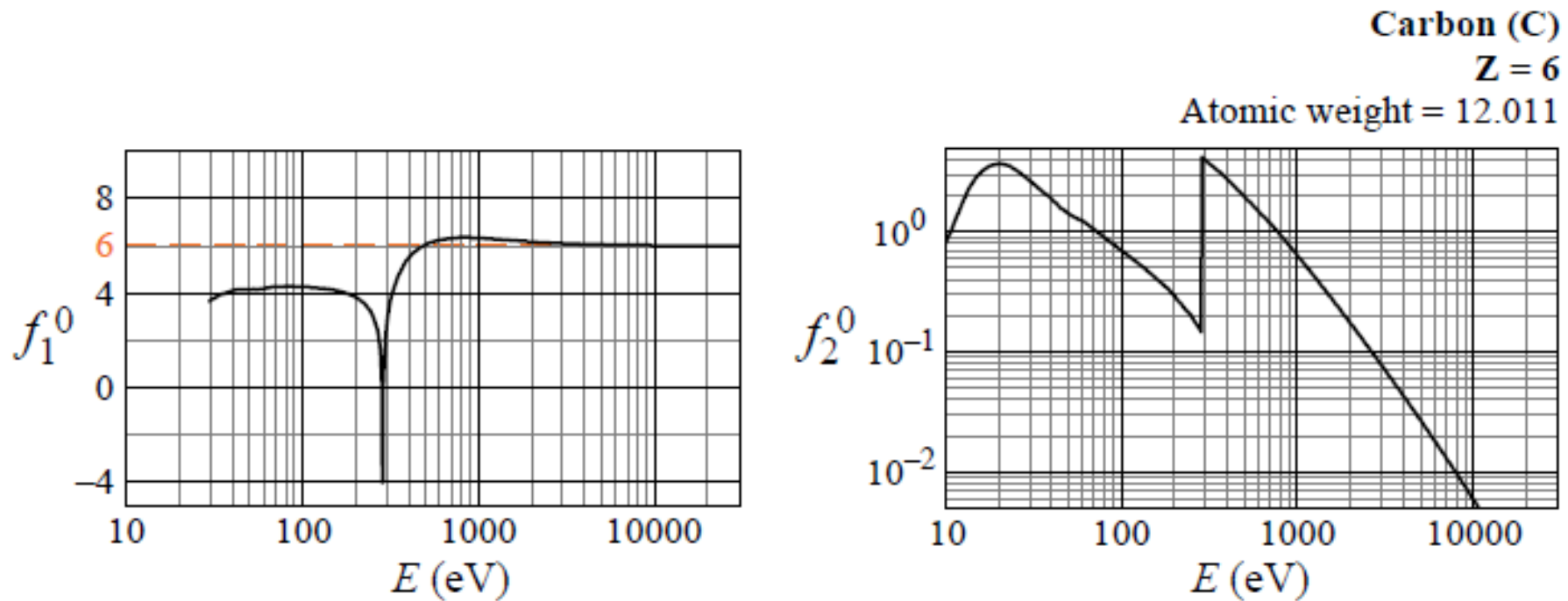
$$\beta = \frac{n_a r_e \lambda^2}{2\pi} f_2^0(\omega) \quad (3.13b)$$



- λ^2 behavior
- δ & $\beta \ll 1$
- δ -crossover

Complex atomic scattering factors

$$f^0(\omega) = f_1^0(\omega) - i f_2^0(\omega)$$



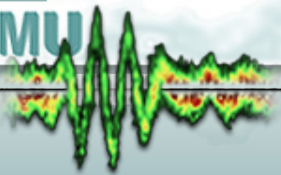
(Henke and Gullikson; [www-cxro.LBL.gov](http://www-cxro.lbl.gov))

valid only for : long wavelength $\lambda \gg a_0$ (0.529 Å) or forward scattering

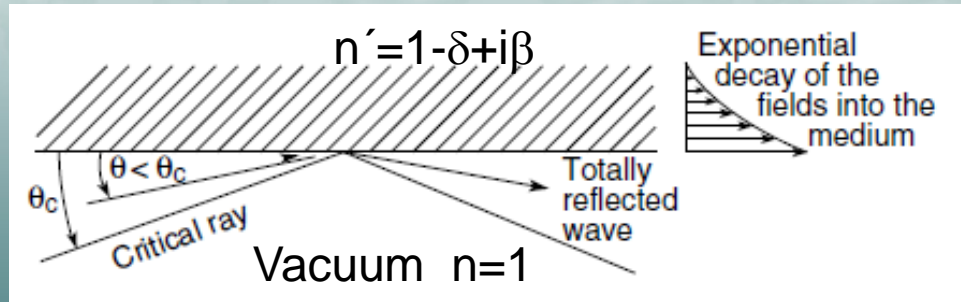
f_1 equals Z for $\omega \gg \omega_s$ all electrons scatter in phase !

Scattering cross section $\sim Z^2$

Single interface optics (example Si)



S- polarisation

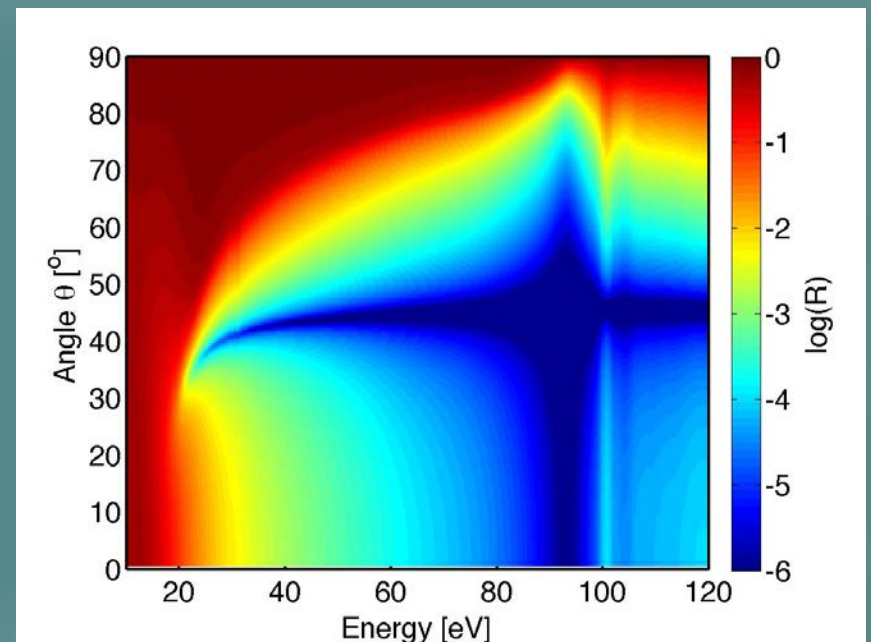
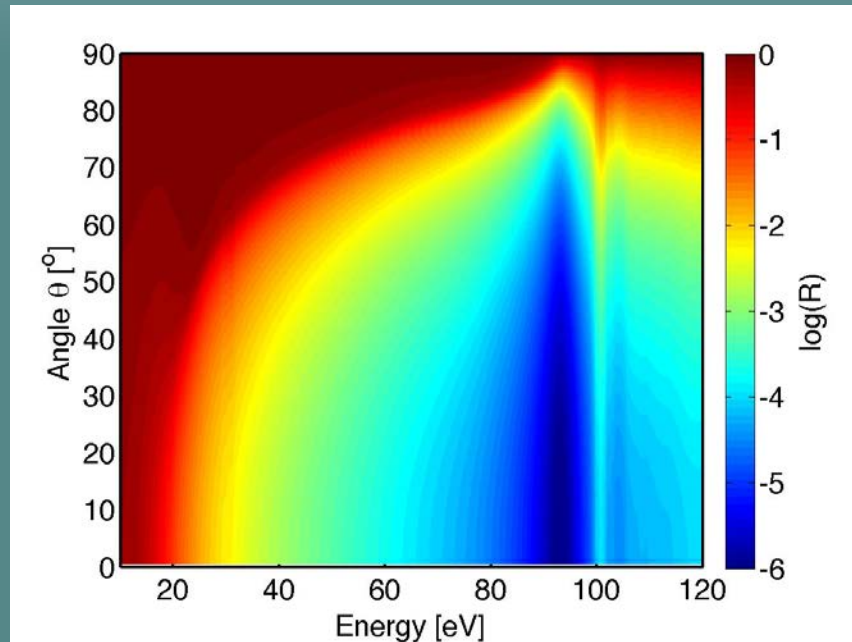


Snell's law :

$$\sin \phi' = \sin \phi / n'$$

Critical angle : $\Theta_c = 2\delta^{1/2}$

P- polarisation



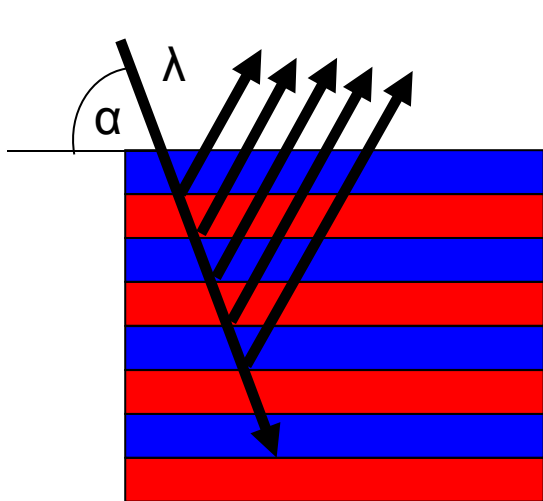
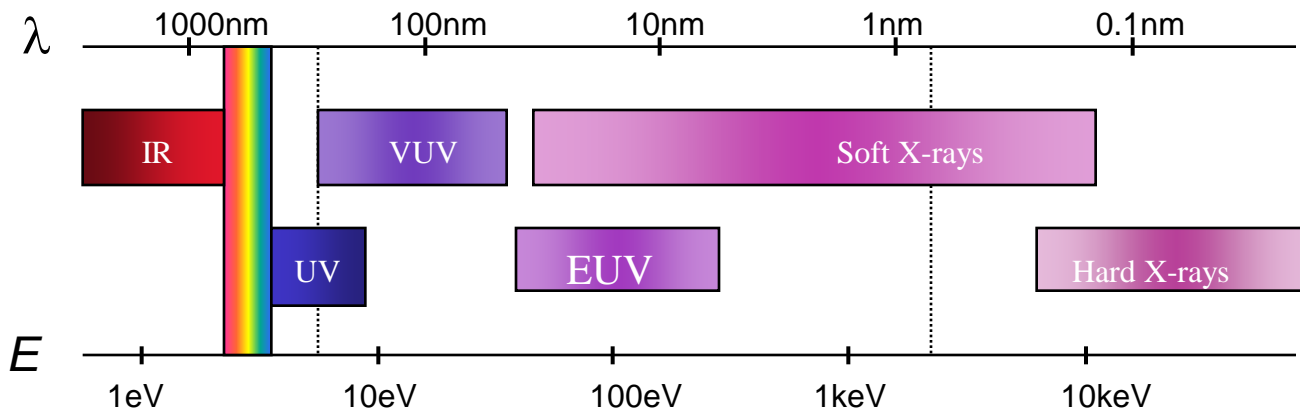
$$R_s = \frac{|\cos \phi - \sqrt{n^2 - \sin^2 \phi}|^2}{|\cos \phi + \sqrt{n^2 - \sin^2 \phi}|^2}$$

Fresnel Equations (s, p pol.)

$$R_{\text{perp}} = (\delta^2 + \beta^2)/4$$

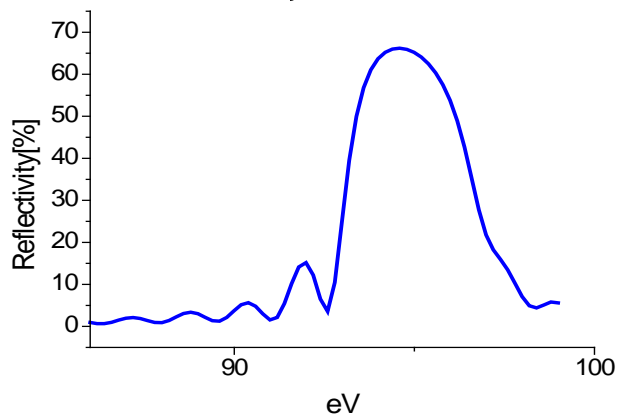
$$R_p = \left| \frac{E_0''}{E_0} \right|^2 = \frac{|n^2 \cos \phi - \sqrt{n^2 - \sin^2 \phi}|^2}{|n^2 \cos \phi + \sqrt{n^2 - \sin^2 \phi}|^2}$$

Principle of XUV multilayer mirrors at near normal incidence angles

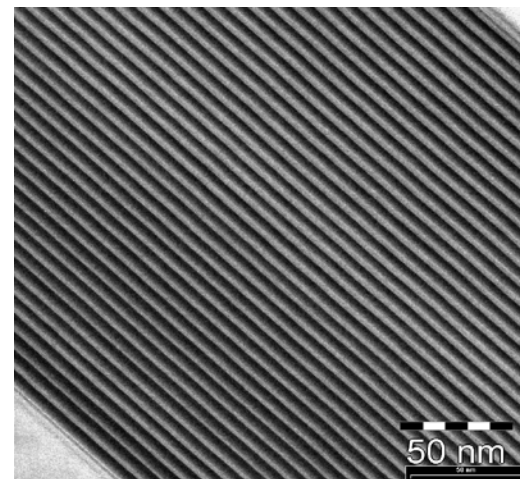


Principle of an XUV multilayer mirror

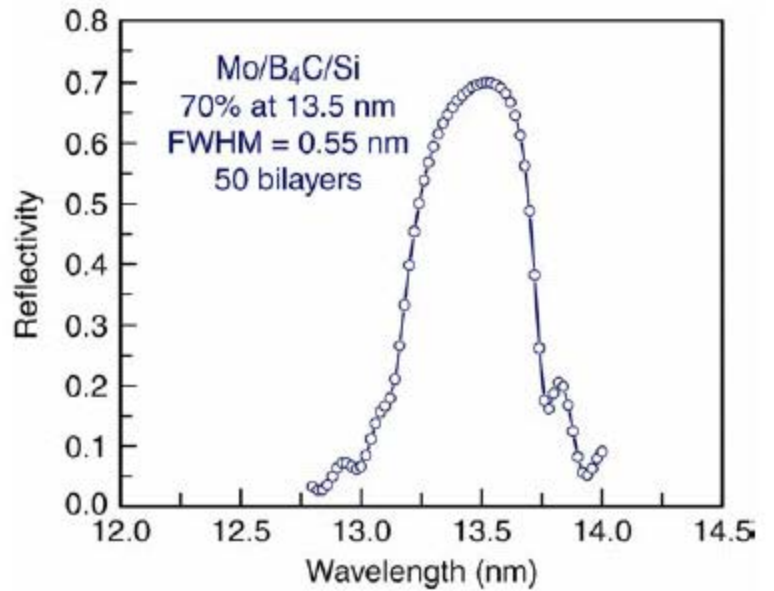
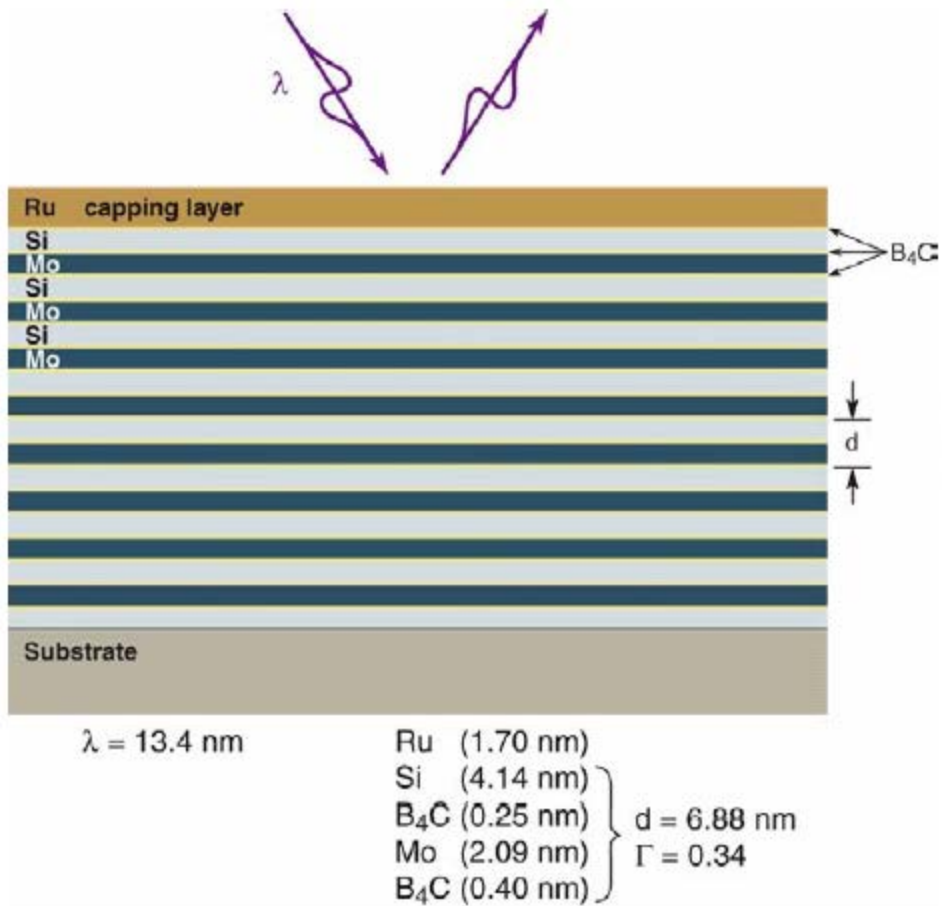
$$m\lambda = 2d \cos \alpha \sqrt{1 - \frac{2\delta - \delta^2}{\cos^2 \alpha}}$$



68% measured reflectivity of a MoSi multilayer mirror @ 93 eV



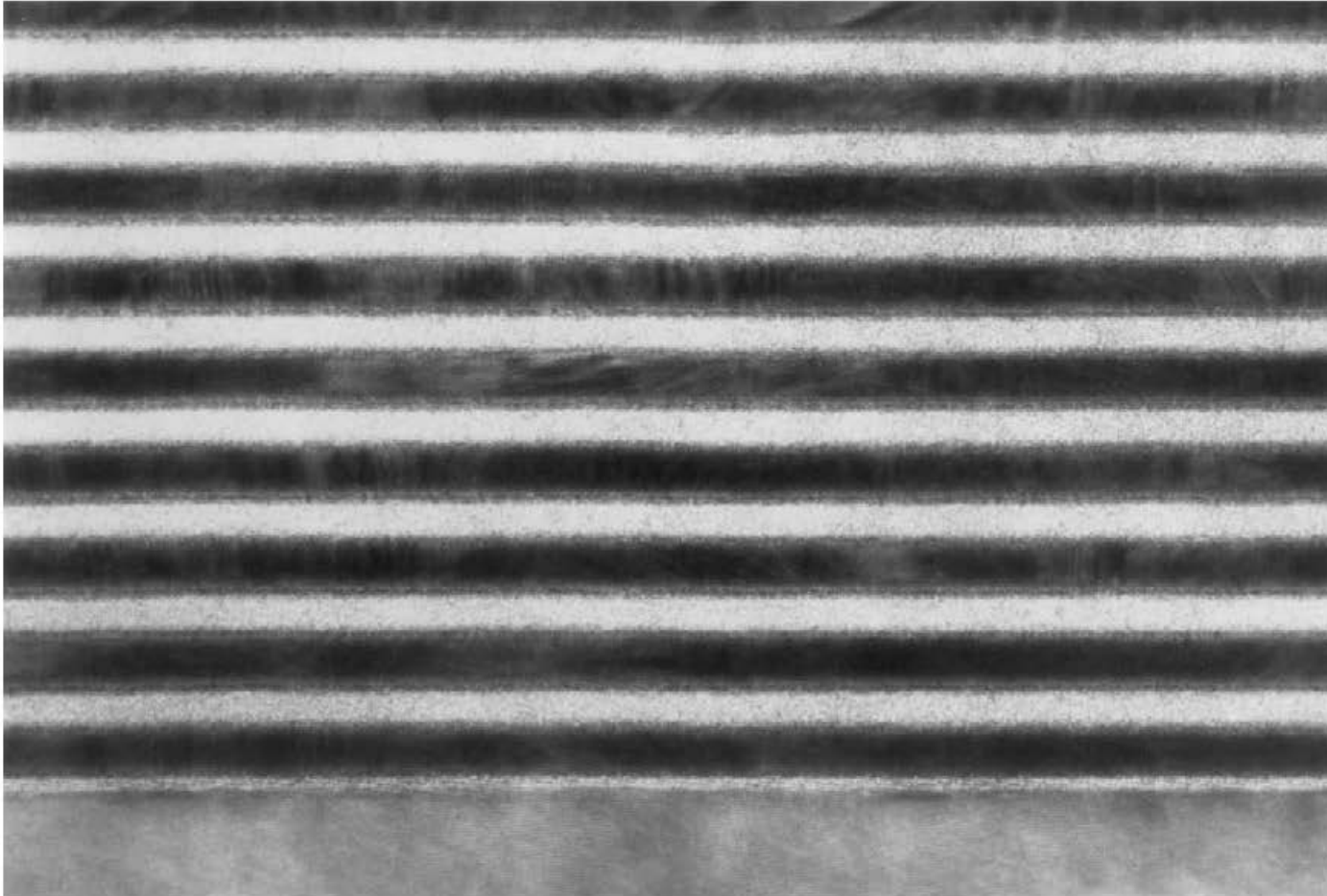
TEM image of a MoSi multilayer



Courtesy of Saša Bajt / LLNL



Mo/Si Multilayer Coating



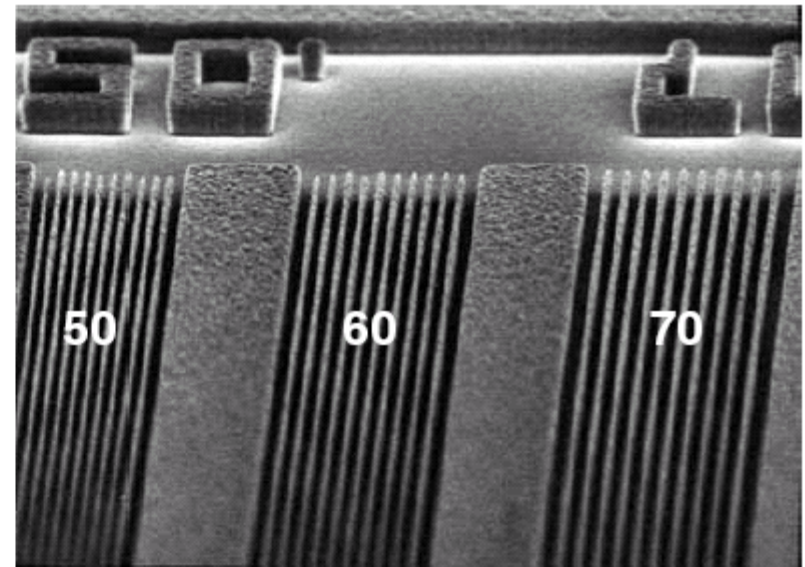
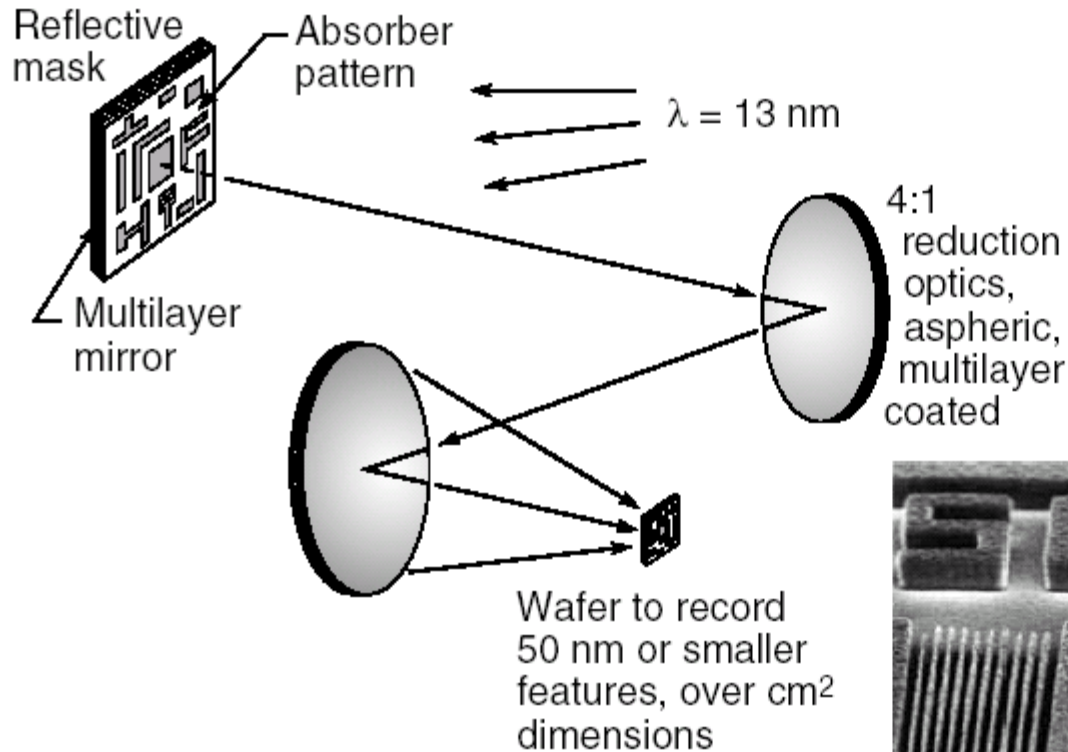
(T. Nguyen, CXRO/LBNL)

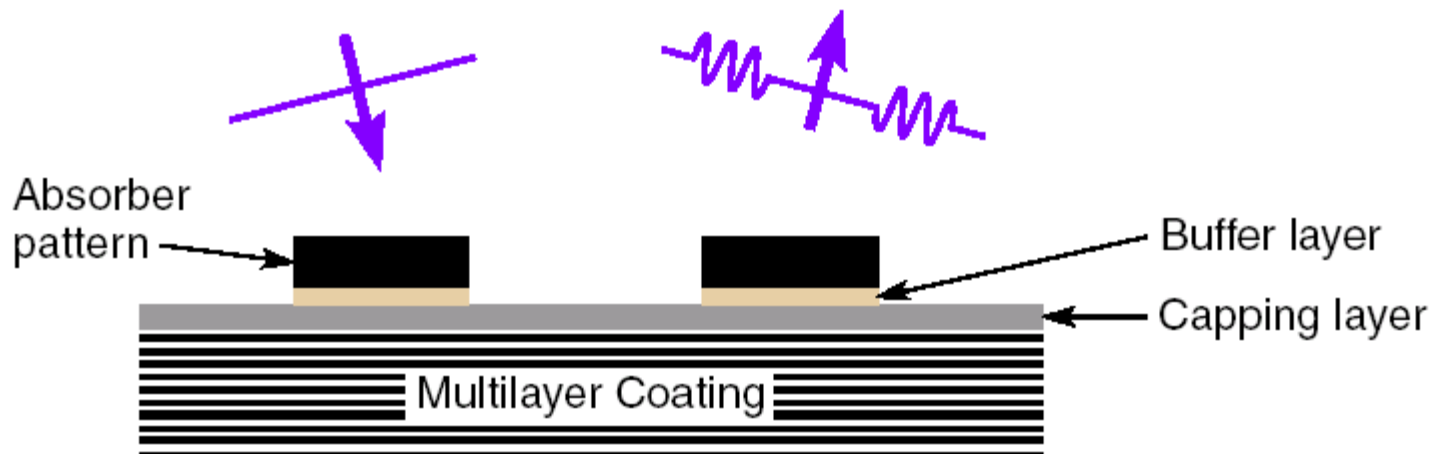
First year of volume production	2001	2003* 2004	2005* 2007	2007* 2010	2009* 2013	2011* 2016
Technology Generation (Dense lines, printed in resist)	130 nm	90 nm	65 nm	45 nm	32 nm	23 nm
Isolated Lines (in resist) [Physical gate, post-etch]	90 nm [65 nm]	53 nm [37 nm]	35 nm [25 nm]	25 nm [18 nm]	18 nm [13 nm]	13 nm [9 nm]
Chip Frequency	1.7 GHz	4.0 GHz	6.8 GHz	12 GHz	19 GHz	29 GHz
Transistors per chip (HV) (3 × for HP ; 5 × for ASICs)	100 M	190 M	390 M	780 M	1.5 B	3.1 B
DRAM Memory (bits)	510 M	1.1 G	4.3 G	8.6 G	34 G	69 G
Gate CD Control (3σ, post-etch)	5 nm	3 nm	2 nm	1.5 nm	1.1 nm	0.7 nm
Field Size (mm × mm)	25 × 32	25 × 32	22 × 26	22 × 26	22 × 26	22 × 26
Chip Size (mm) (2.2 × for HP ; to 4 × for ASIC)	140	140	140	140	140	140
Water Size (diameter)	300 mm	300 mm	300 mm	450 mm	450 mm	450 mm

*Semiconductor Industry Association (SIA), December 2001. *Possible 2-year cycle.



Extreme Ultraviolet (EUV) Lithography





Substrate:
Low thermal expansion material (LTEM)
(6" square \times 1/4" thick)

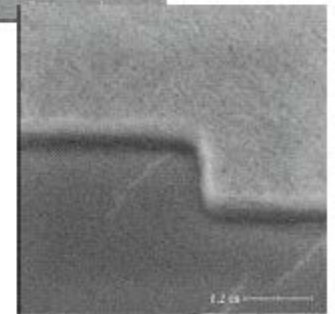
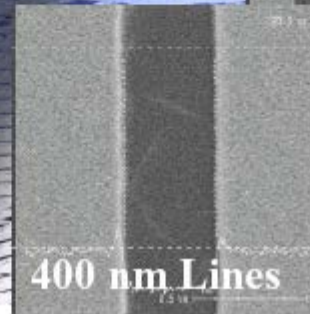
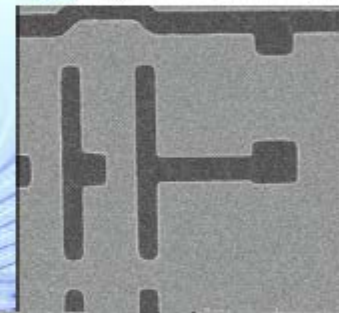
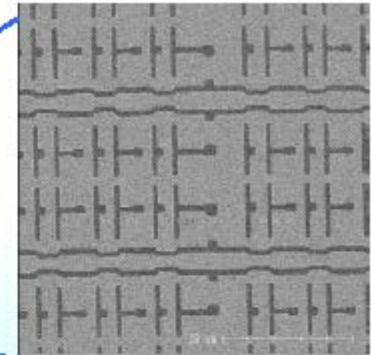
Typically

Mo/Si multilayer ($d = 6.7$ nm)
with 30 nm SiO_2 capping layer

Cr or TaN absorber (~ 70 nm)
with 50 nm Ru Buffer layer

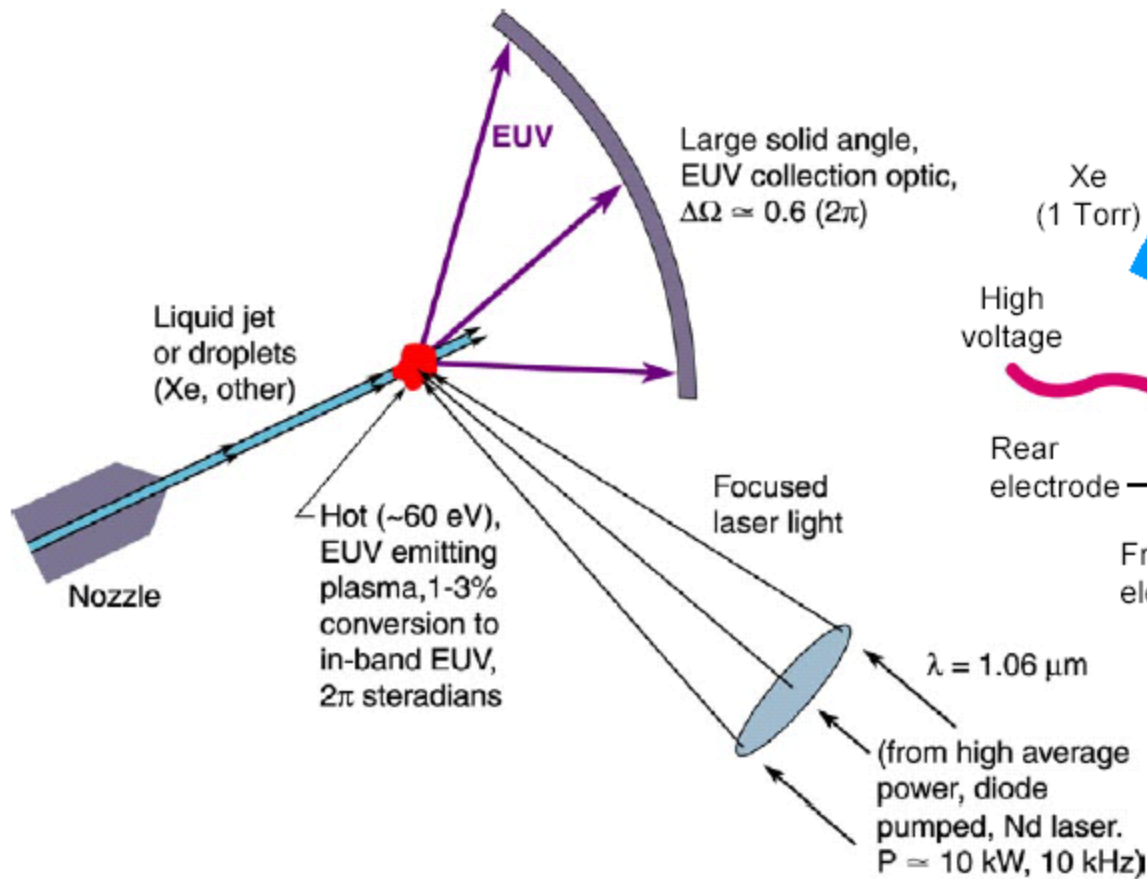
LTEM substrate
(Ti-doped fused silica)
ULE (Corning), or
Zerodur (Schott)

Stack: Cr/SiON/MoSi Multilayers

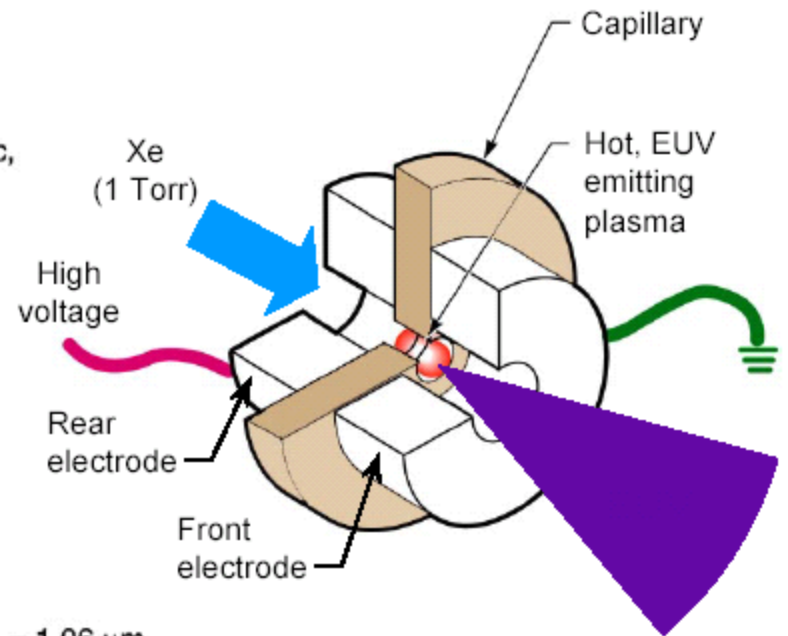


- CD Uniformity +/- 10% Target
 - Data set for 100 nm node
- Min. Resolution: 0.32 μm

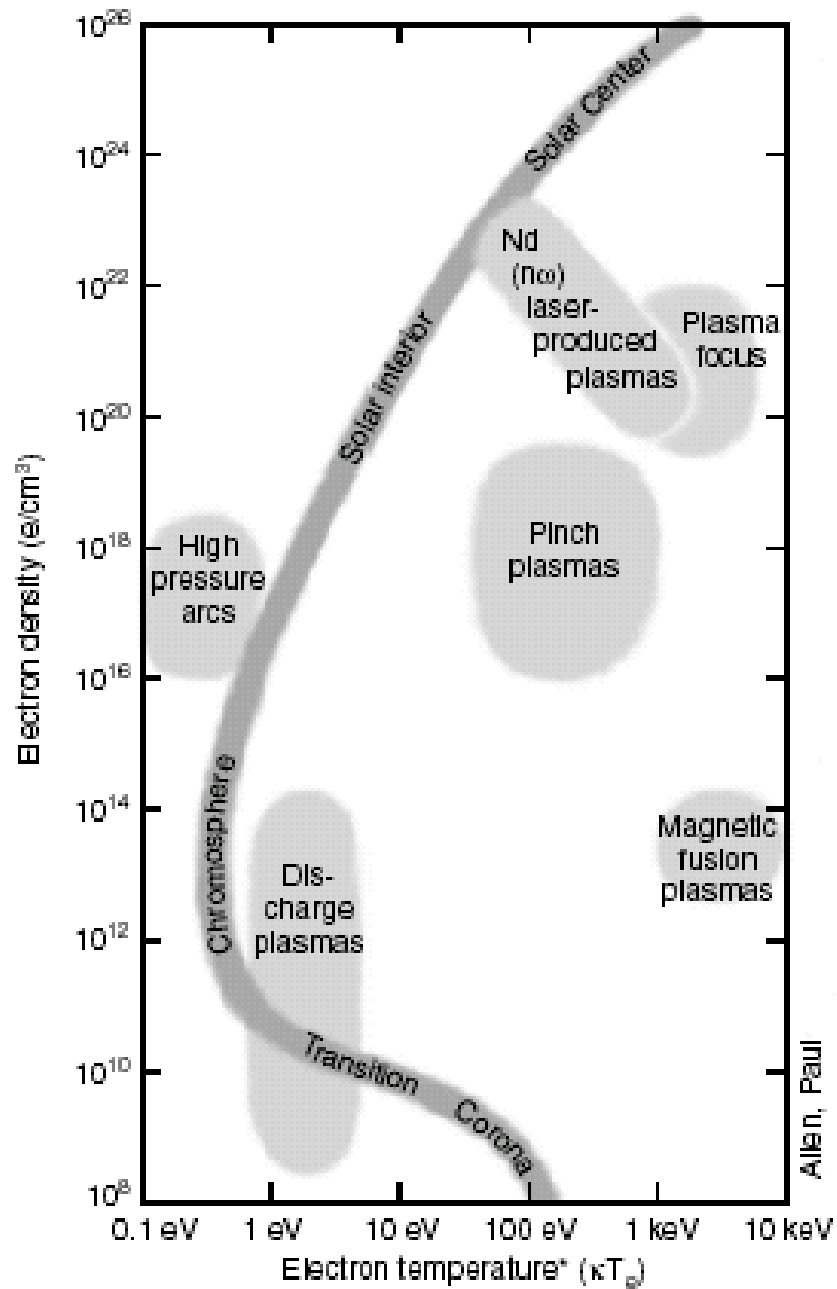
Laser Produced Plasma Source



Electrical Discharge Plasma Source

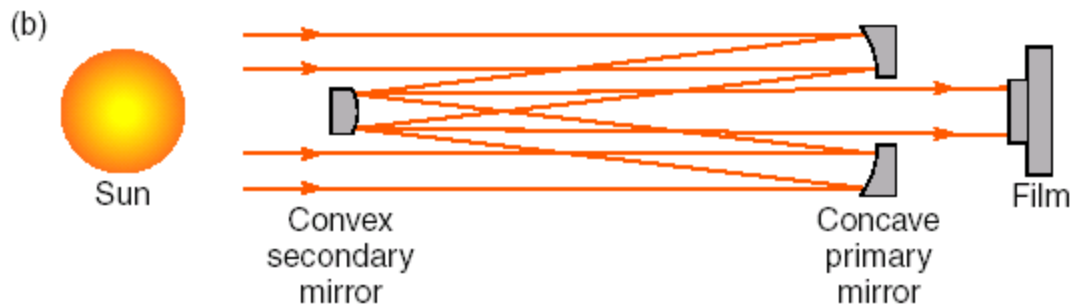
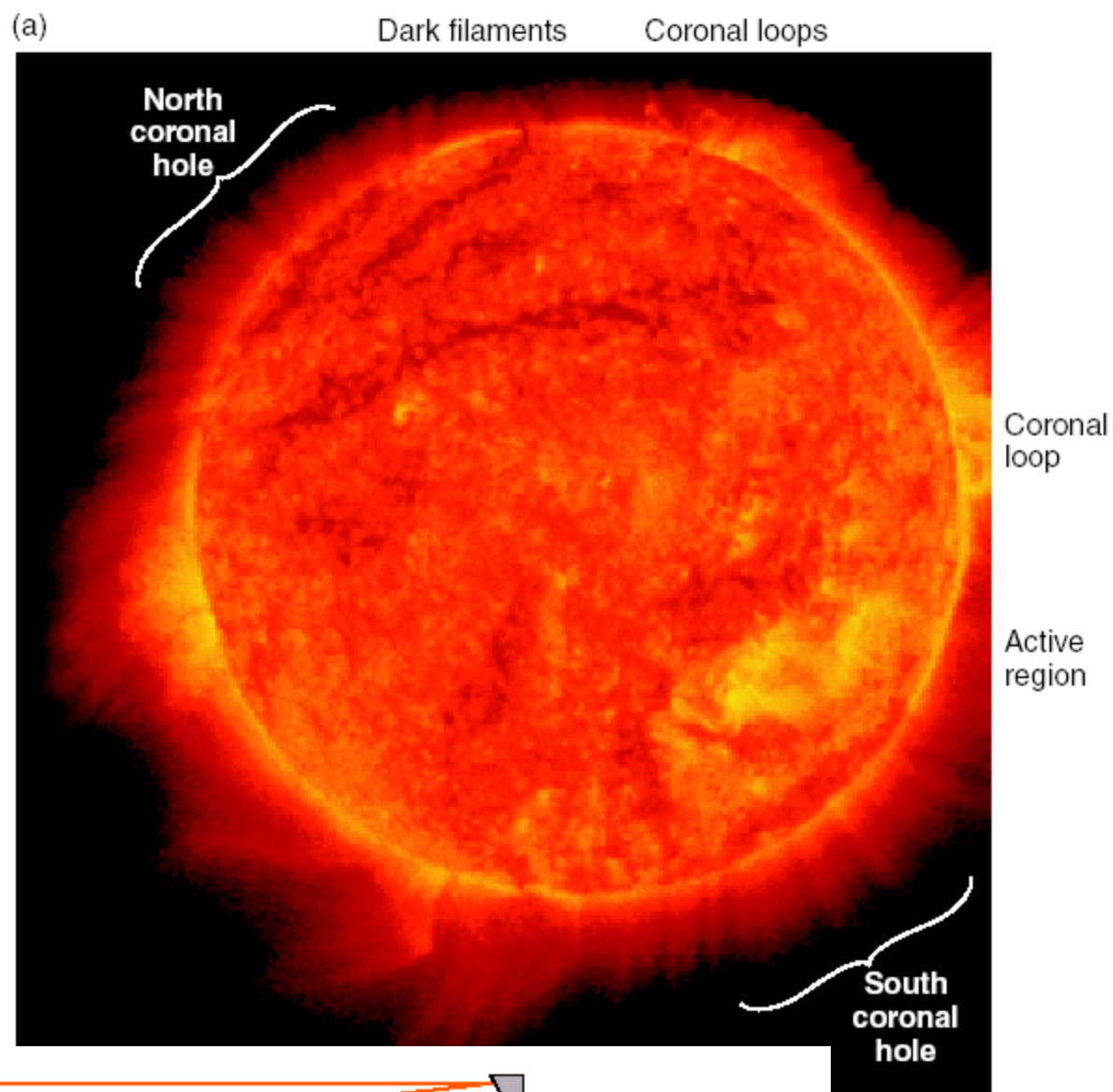


Courtesy of Neil Fornaciari and Glenn Kubiak, Sandia.

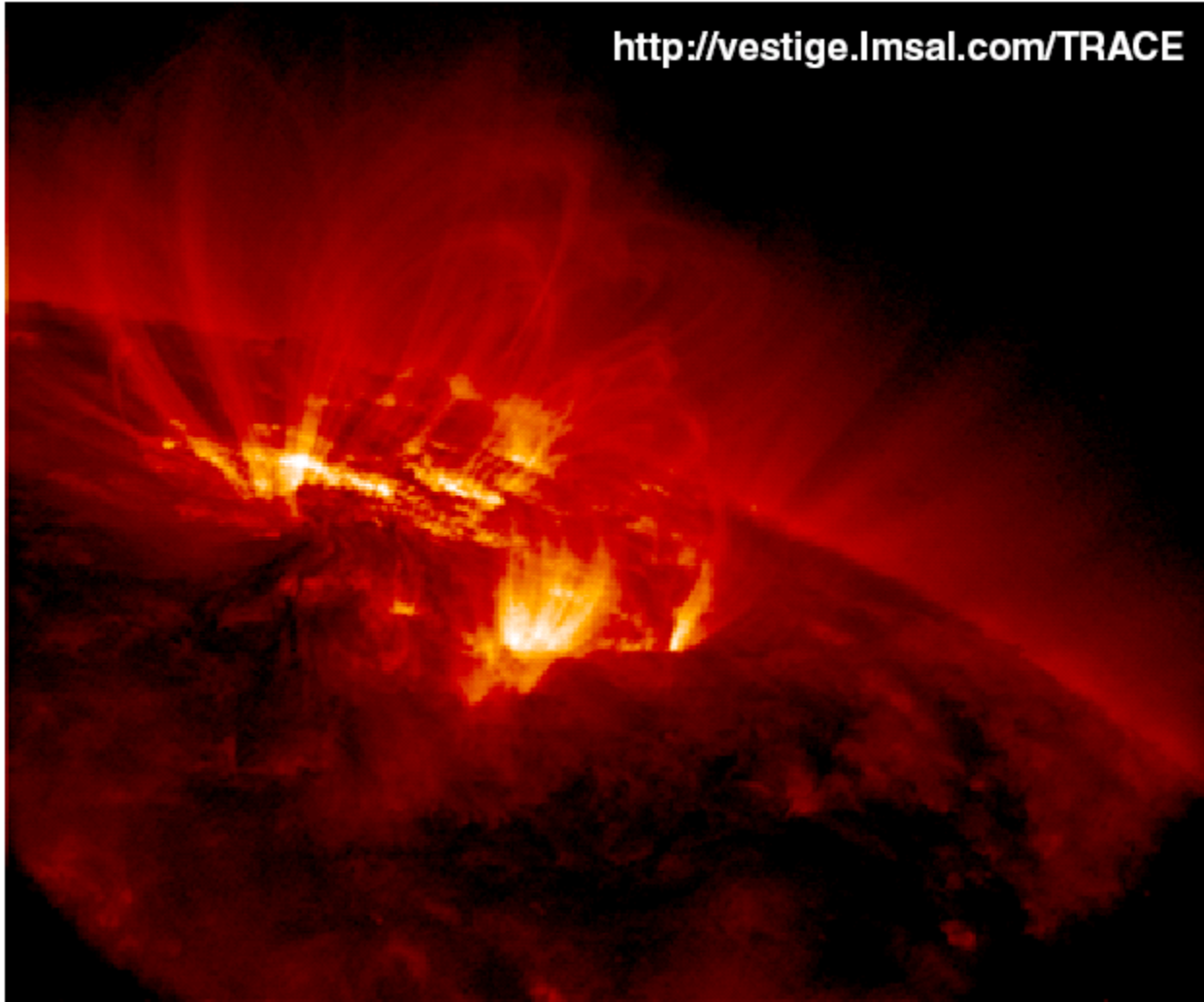


Allen, Paul

EIT telescope
SOHO mission

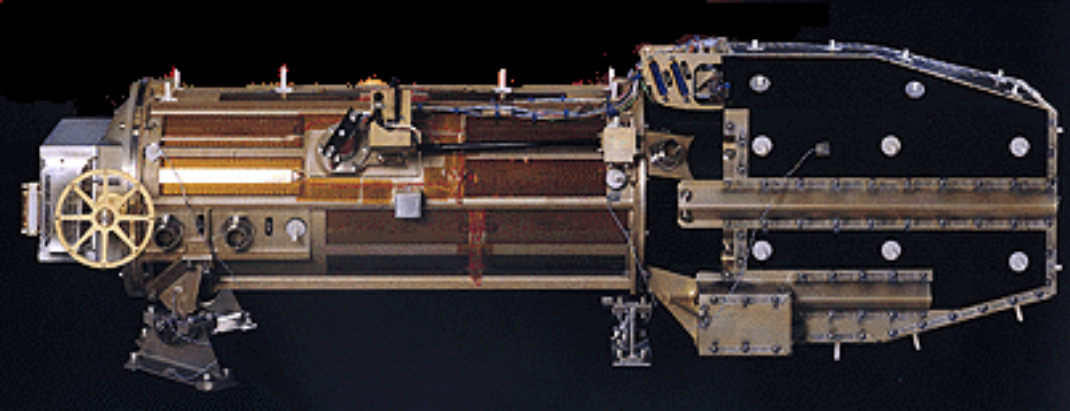


<http://vestige.lmsal.com/TRACE>

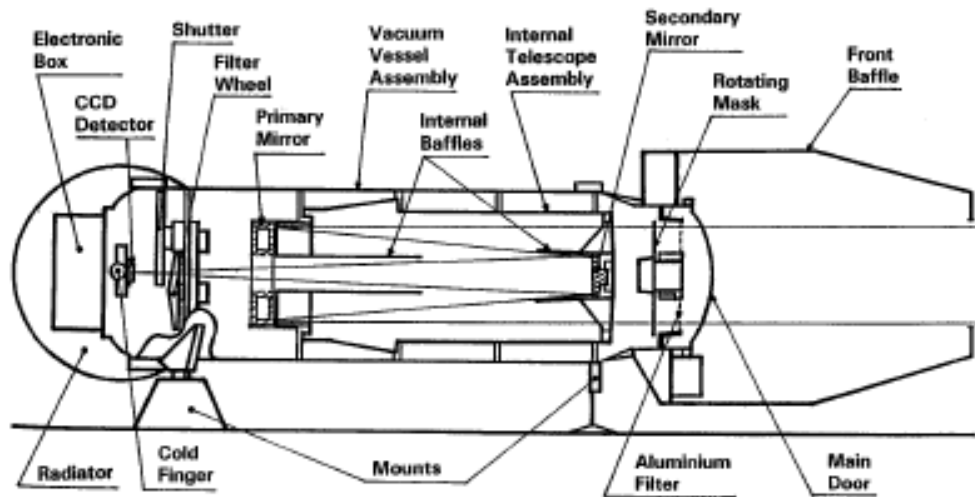
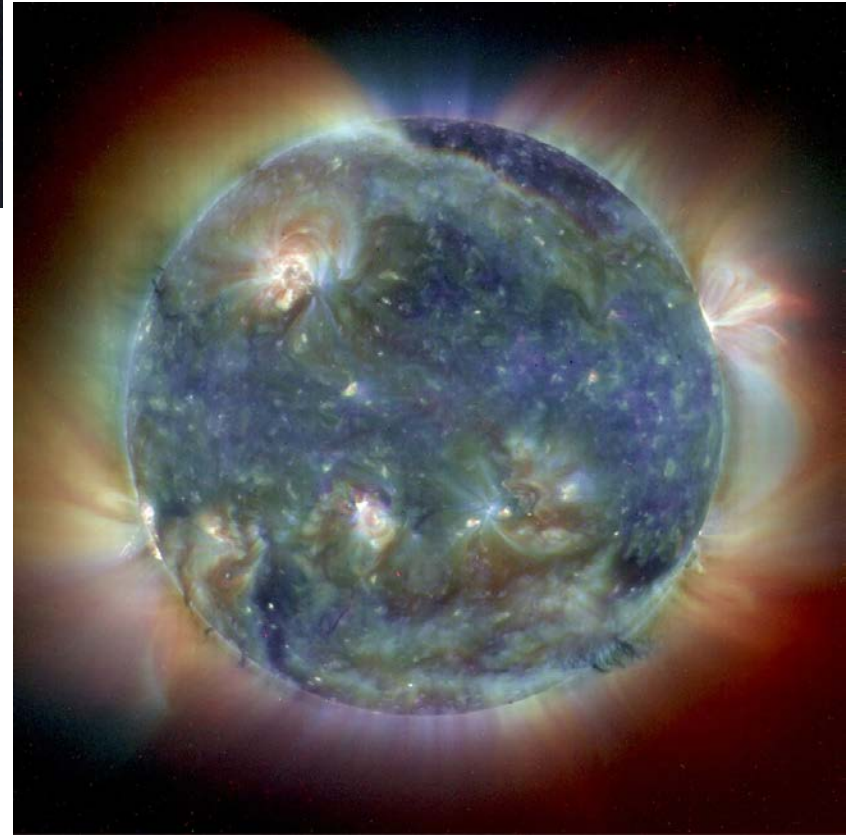


(Courtesy of L. Golub, Harvard-Smithsonian and T. Barbee, LLNL)

The Extreme Ultraviolet Imaging Telescope (EIT)



EIT composite image



Multilayer coated normal incidence cassegrain objective

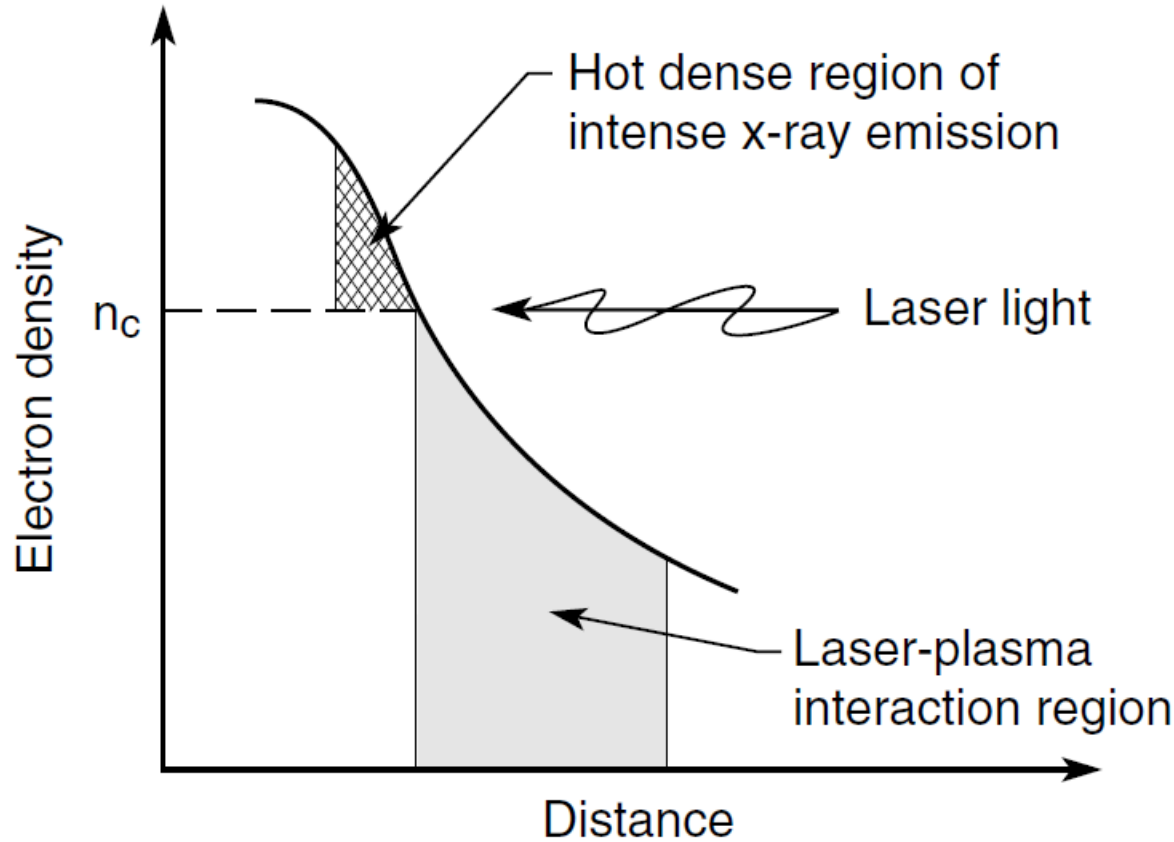
17,1 nm (blue)

19.5 nm (green)

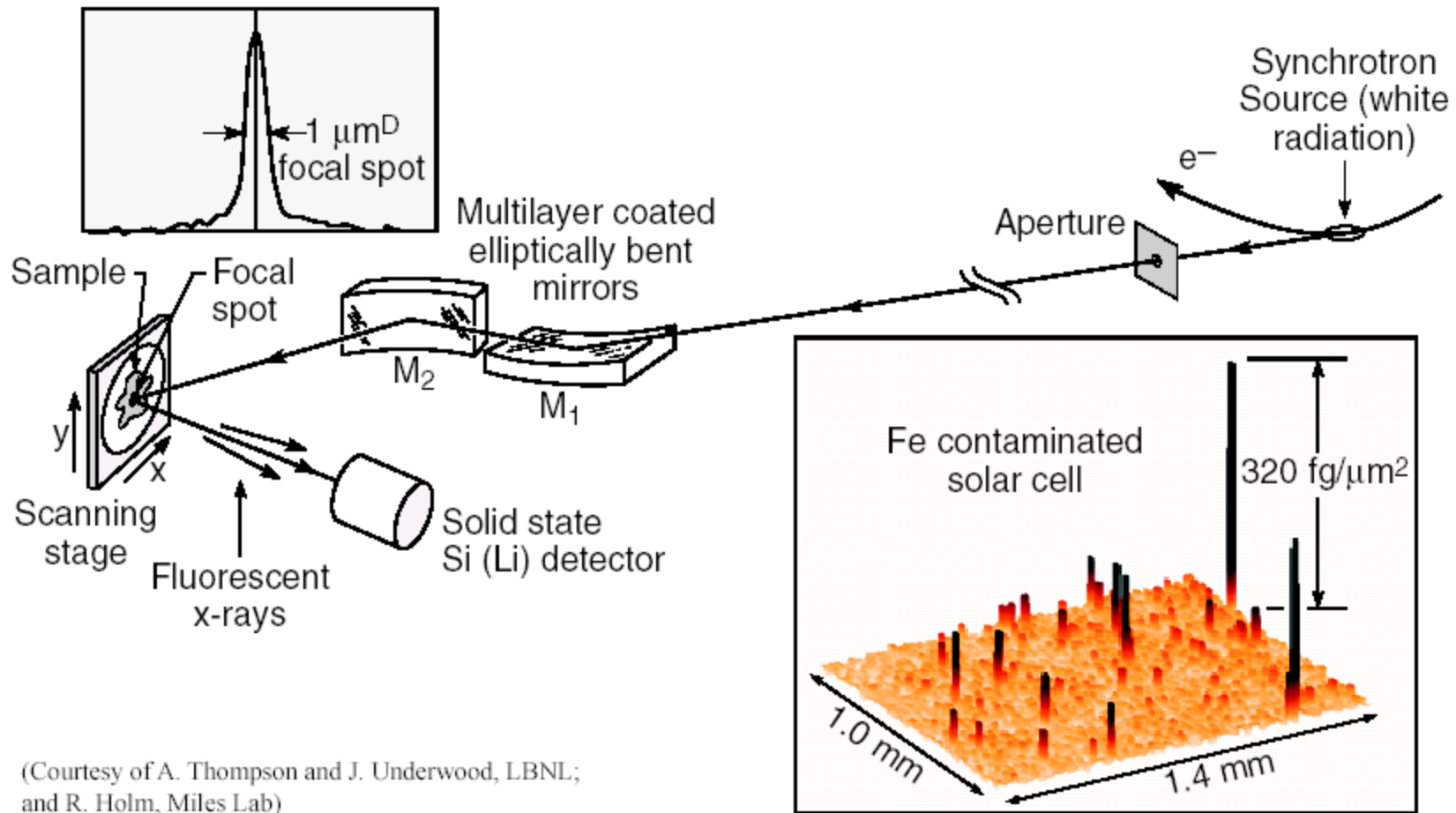
28,4 nm (red)



Soft X-Ray/EUV Emission from a Hot-Dense Plasma

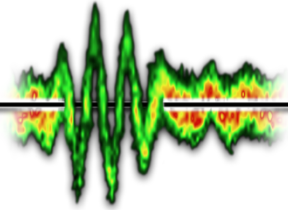


- $kT_e \sim 50 \text{ eV to } 1 \text{ keV}$
- $n_e \sim 10^{20} \text{ to } 10^{22} \text{ e/cm}^3$



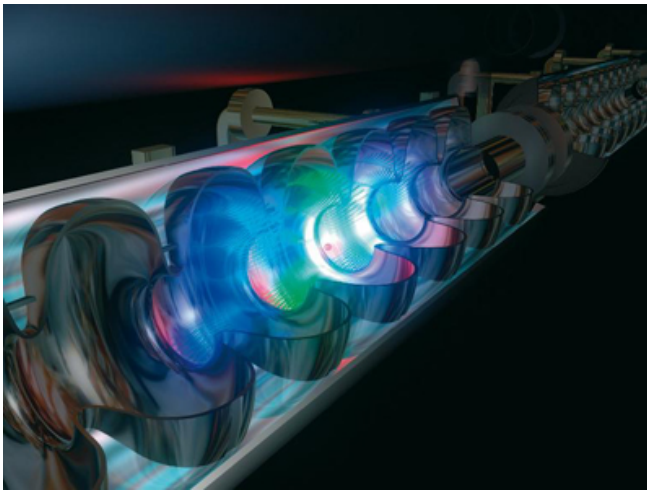
(Courtesy of A. Thompson and J. Underwood, LBNL; and R. Holm, Miles Lab)

Motivation



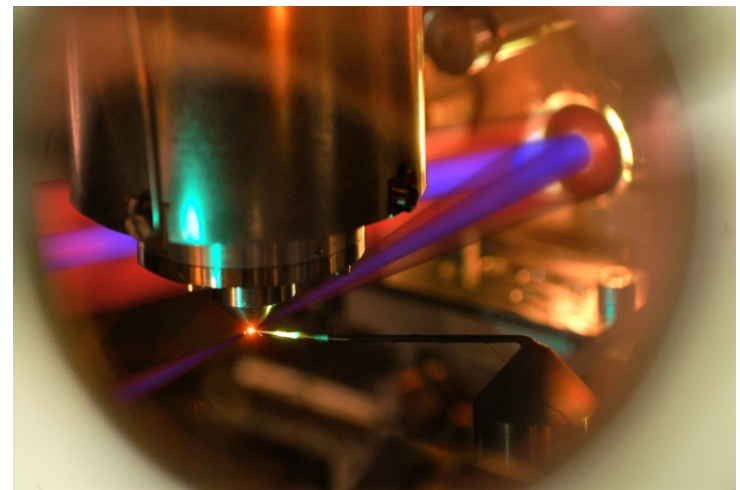
Excellent XUV optics for sources emitting ultrashort pulses:

FEL



(aspiration for sub-) fs pulses
grazing optics due to high intensities

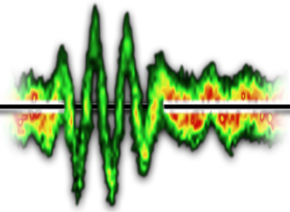
HHG



as pulses (requires large ΔE)
normal incidence optics possible

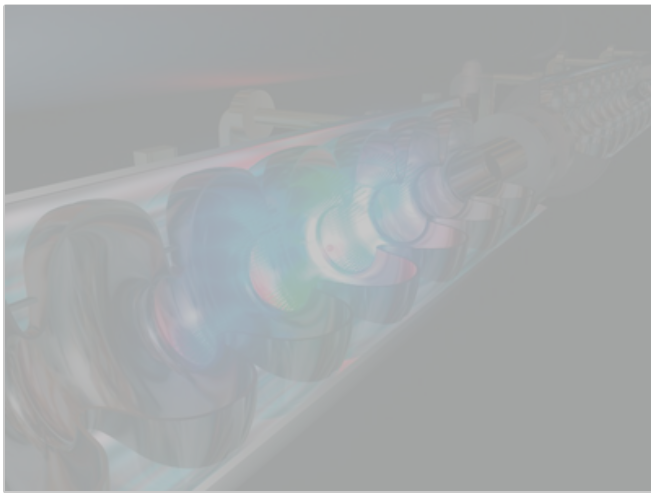
Both require optics for spectral filtering, phase shaping, ...

Motivation



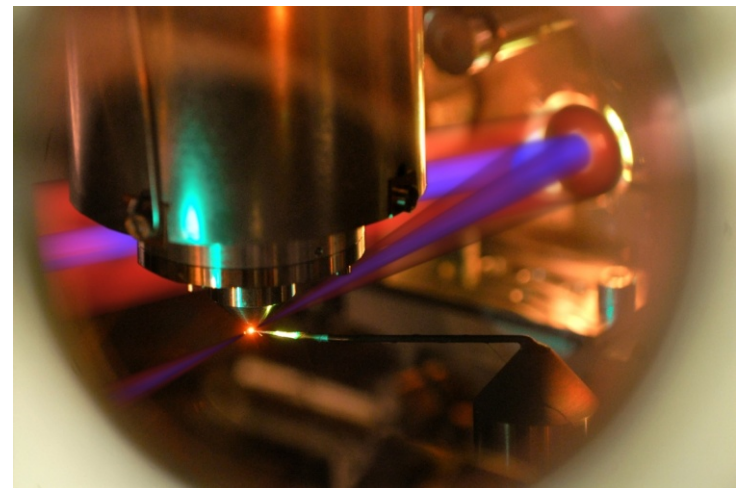
Excellent XUV optics for sources emitting ultrashort pulses:

FEL



(aspiration for sub-) fs pulses
grazing optics due to high intensities

OUR FOCUS: HHG

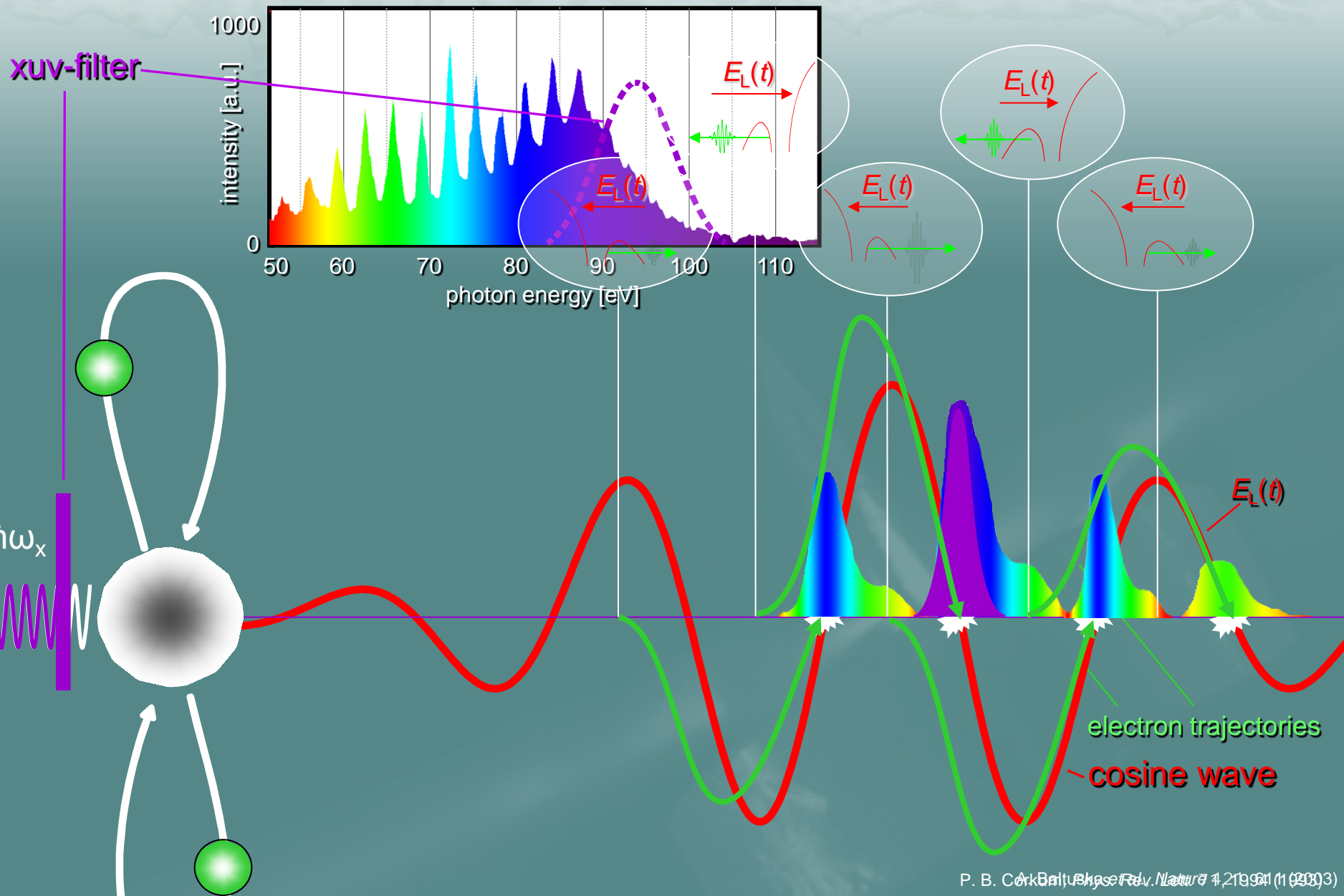


as pulses (requires large ΔE)
normal incidence optics possible

Photon flux essential!

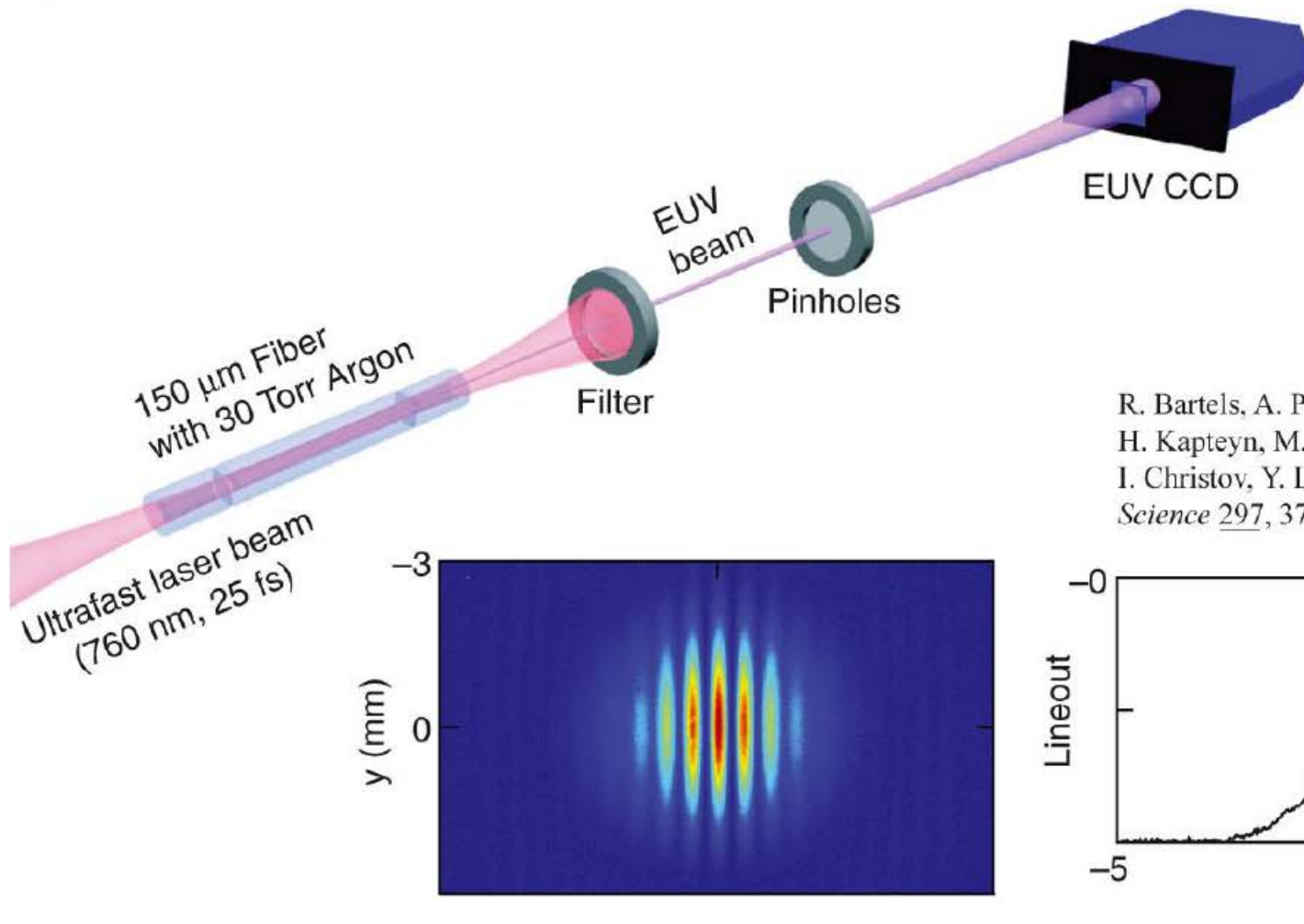
Both require optics for spectral filtering, phase shaping, ...

steering bound electrons with controlled light fields: the birth of an attosecond pulse





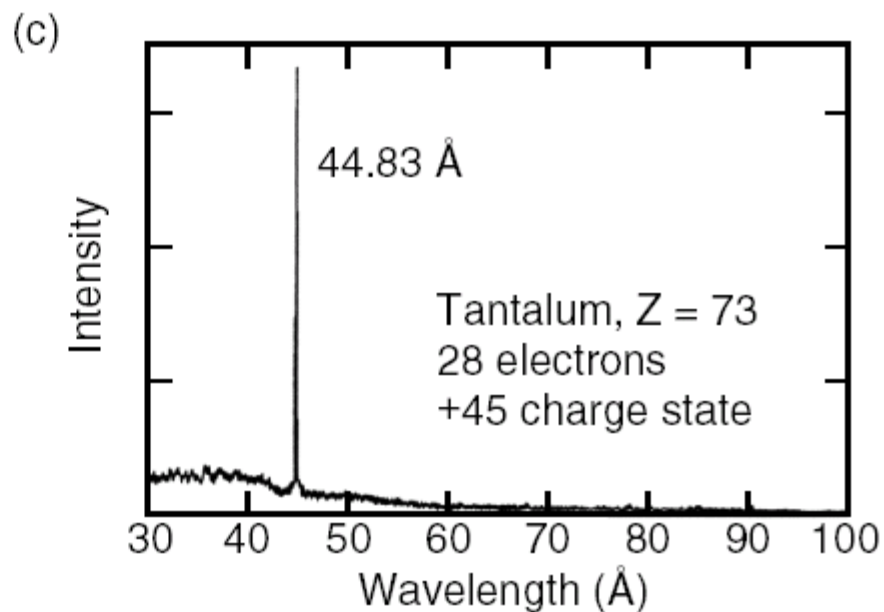
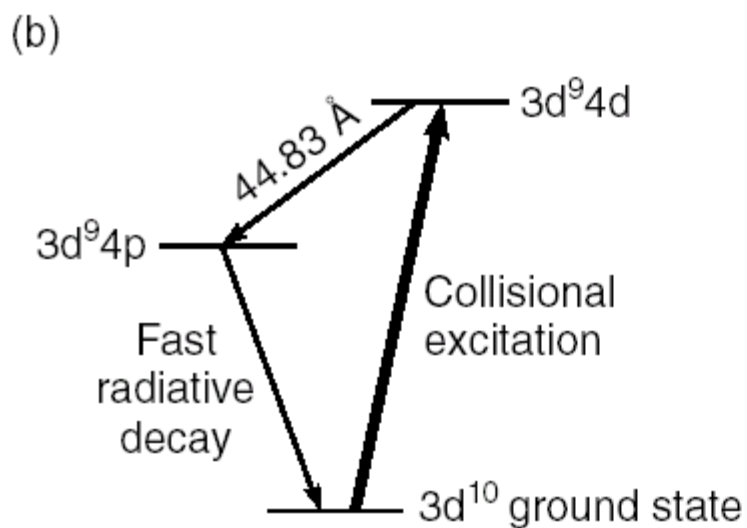
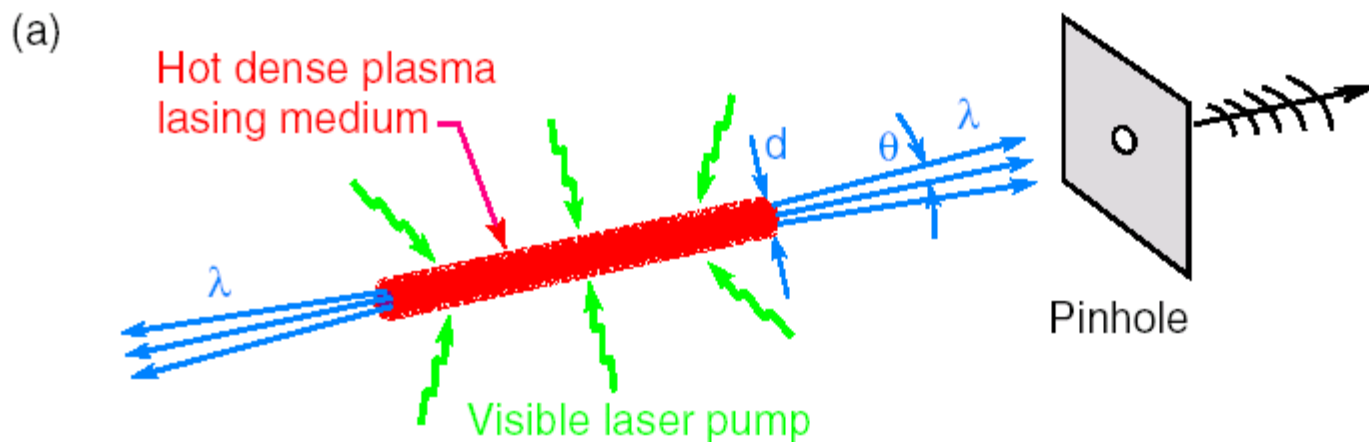
High Harmonic Generation (HHG) Provides Coherent, Femtosecond Pulses



R. Bartels, A. Paul, H. Green,
H. Kapteyn, M. Murnane, S. Backus,
I. Christov, Y. Liu, D. Attwood, C. Jacobsen,
Science 297, 376 (19 July 2002).

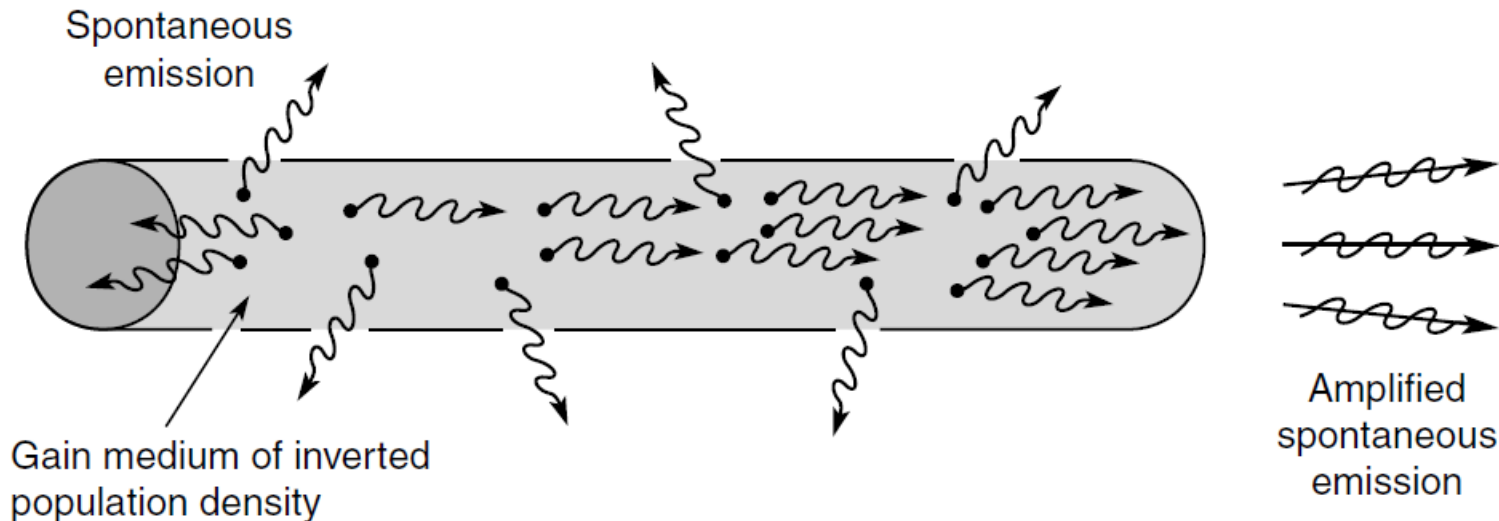
$$P \approx 10 \mu\text{W} \rightarrow 2 \times 10^{12} \text{ ph/sec @ } 36 \text{ nm } (n = 21; 34 \text{ eV})$$

Courtesy of Professors Margaret Murnane and Henry Kapteyn, Univ. Colorado

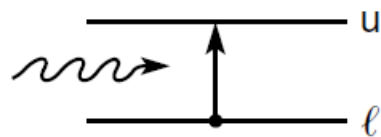




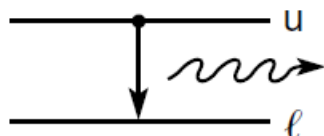
Lasing Begins with Amplified Spontaneous Emission



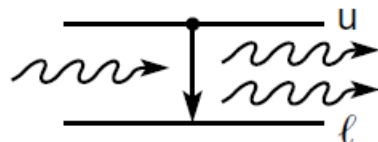
(a) Absorption



(b) Spontaneous emission



(b) Stimulated emission



$$\frac{I}{I_0} = e^{GL} \quad (7.2)$$

$$G = n_u \sigma_{\text{stim}} F \quad (7.4)$$

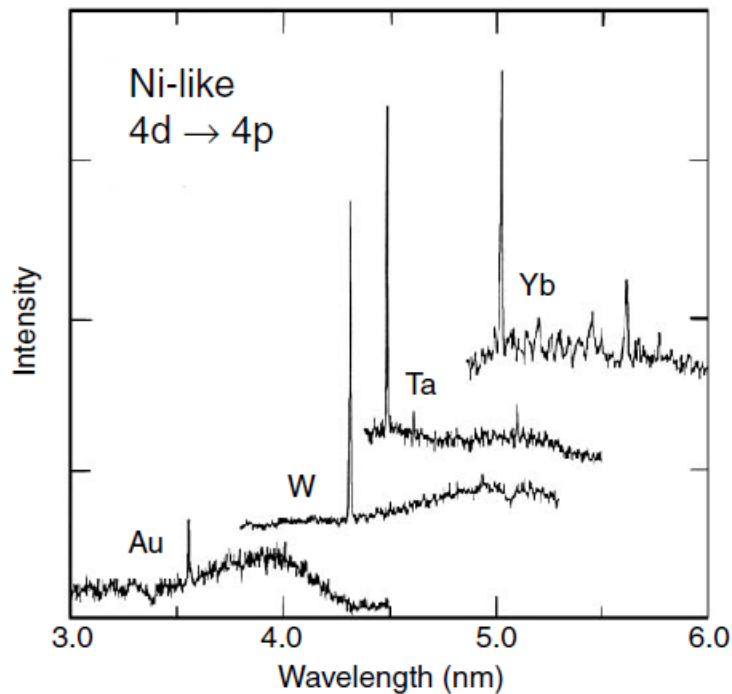
$$\sigma_{\text{stim}} = \frac{\pi \lambda r_e}{(\Delta \lambda / \lambda)} \left(\frac{g_l}{g_u} \right) f_{lu} \quad (7.18)$$

$$\frac{P}{A} = \frac{16\pi^2 c^2 \hbar (\Delta \lambda / \lambda) GL}{\lambda^4} \quad (7.22)$$

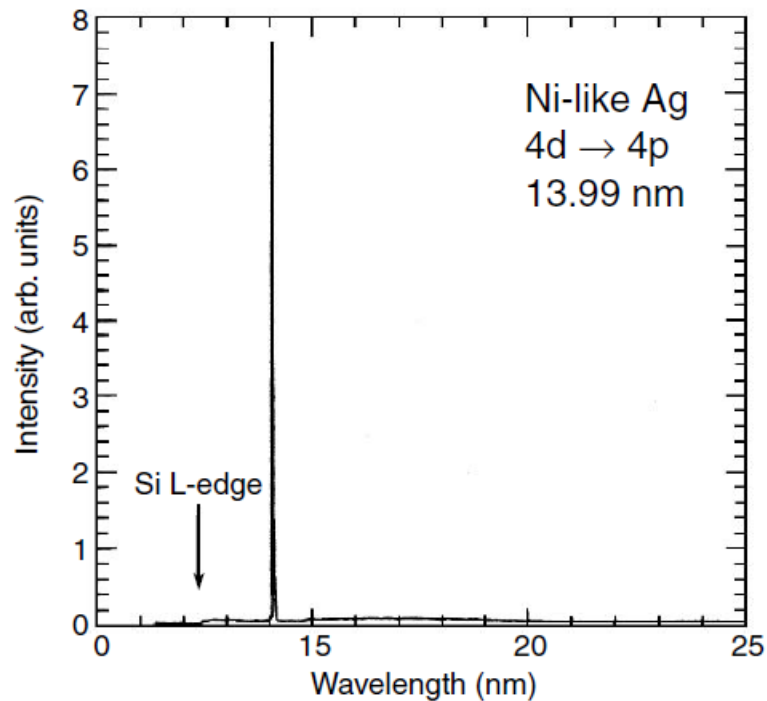


Milli-watt XUV/EUV Lasing (ASE) at Various Labs Around the World

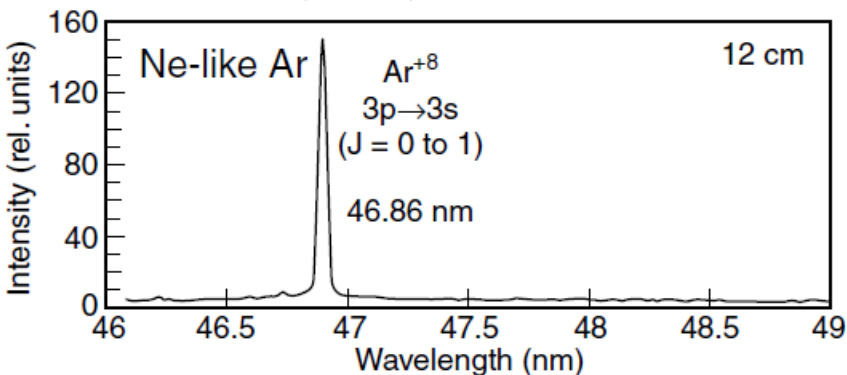
B. MacGowan, et al., Lawrence Livermore Lab



J. Zhang, et al., Rutherford-Appleton



J. Rocca, et al., Colorado State Univ.



S. Suckewer, Princeton Univ.

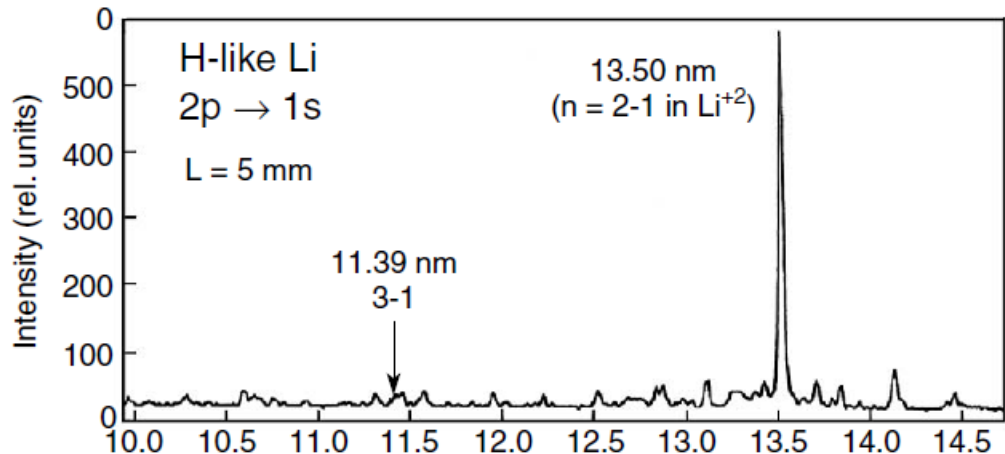
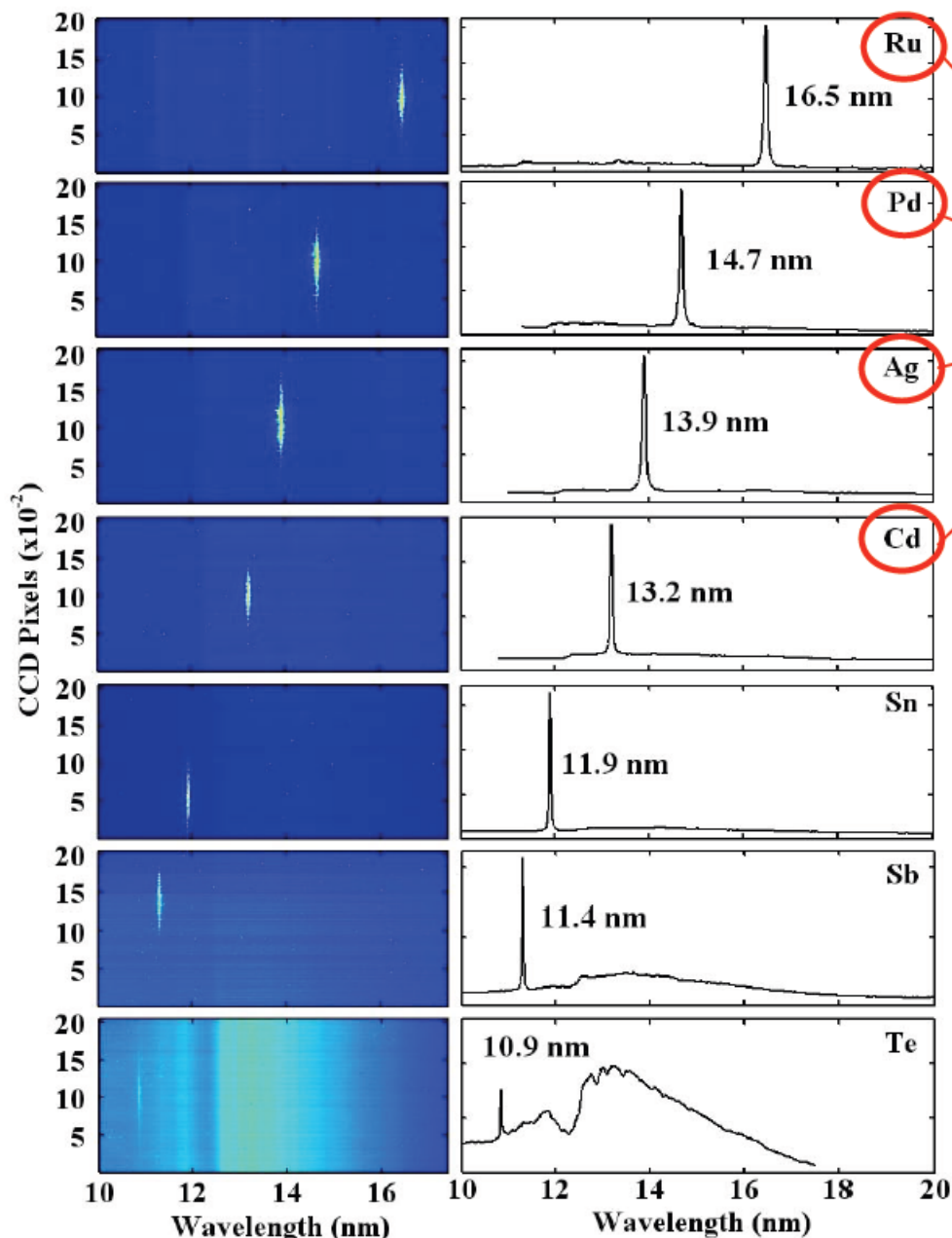


Table-top EUV Lasers

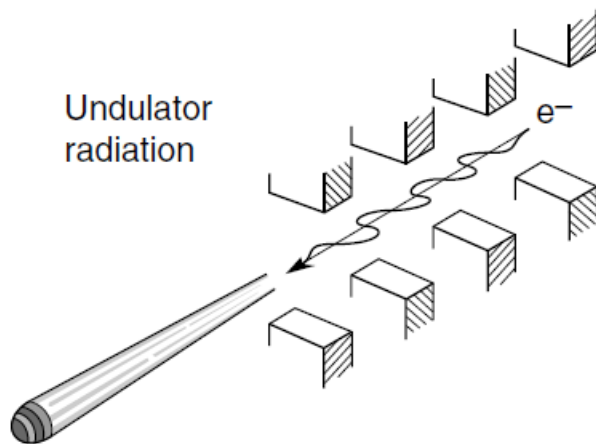
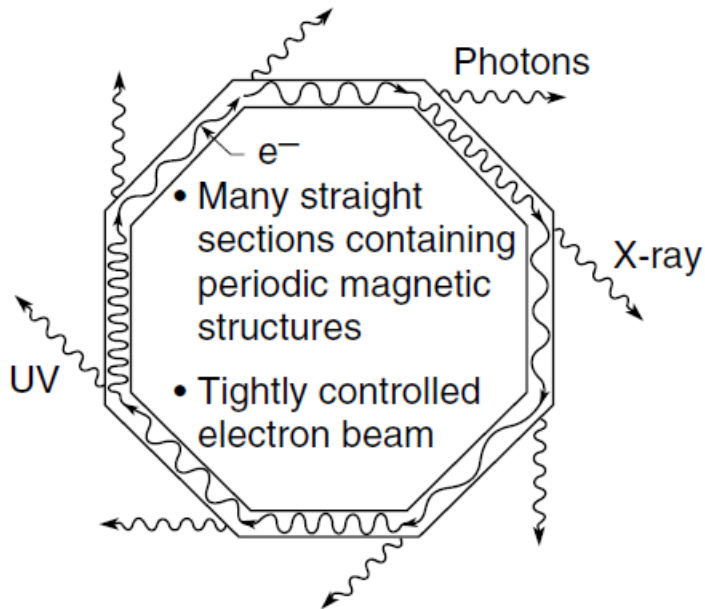


Gain saturated
operation
demonstrated

J.J. Rocca et
al. Presented
at SPIE
Conf. 5919.
San Diego
(2005)



Synchrotron Radiation



Bending Magnet:

$$\hbar\omega_c = \frac{3e\hbar B\gamma^2}{2m} \quad (5.7)$$

Wiggler:

$$\hbar\omega_c = \frac{3e\hbar B\gamma^2}{2m} \quad (5.80)$$

$$n_c = \frac{3K}{4} \left(1 + \frac{K^2}{2}\right) \quad (5.82)$$

$$P_T = \frac{\pi e K^2 \gamma^2 I N}{3\epsilon_0 \lambda_u} \quad (5.85)$$

Undulator:

$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right) \quad (5.28)$$

$$K = \frac{eB_0\lambda_u}{2\pi mc} \quad (5.18)$$

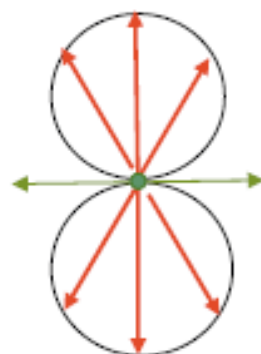
$$\theta_{\text{cen}} = \frac{1}{\gamma^* \sqrt{N}} \quad (5.15)$$

$$\left. \frac{\Delta\lambda}{\lambda} \right|_{\text{cen}} = \frac{1}{N} \quad (5.14)$$

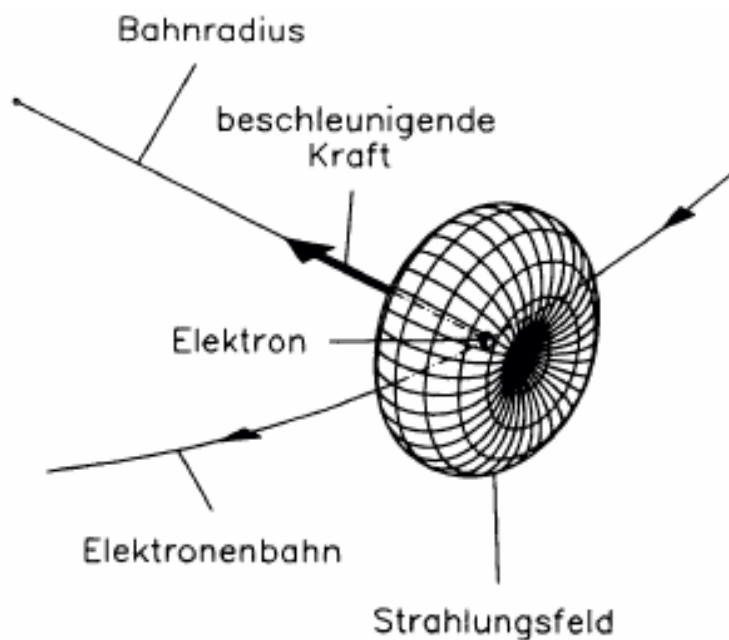
$$\bar{P}_{\text{cen}} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{\left(1 + \frac{K^2}{2}\right)^2} f(K) \quad (5.41)$$

IV.1.2 Synchrotronstrahlungsquellen

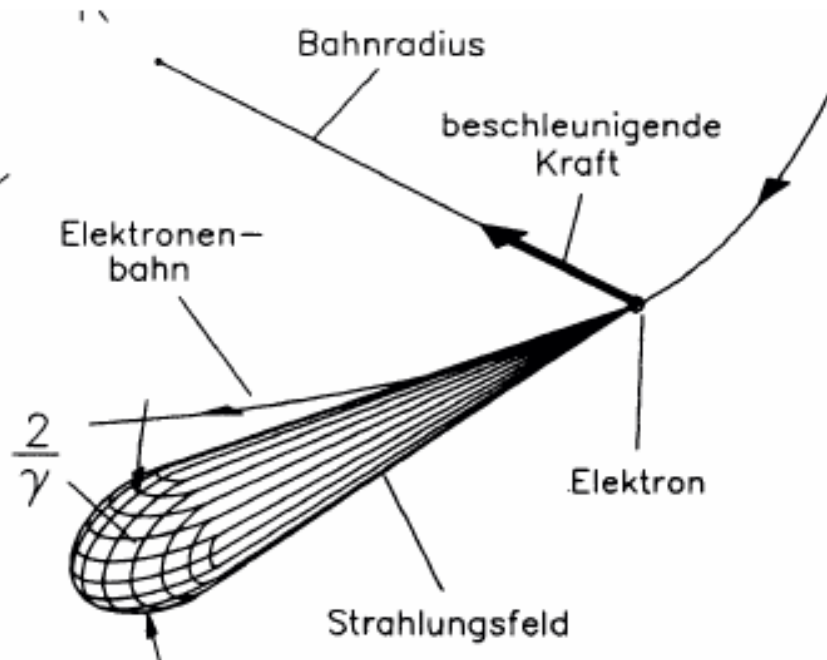
Prinzip: **Abstrahlung** elektromagnetischer Wellen durch **beschleunigte Ladung** (vgl. oszillierender Dipol)



hier: **Querbeschleunigung** der Ladungen bei **relativistischer Geschwindigkeit**



Schwerpunktsystem des Elektrons

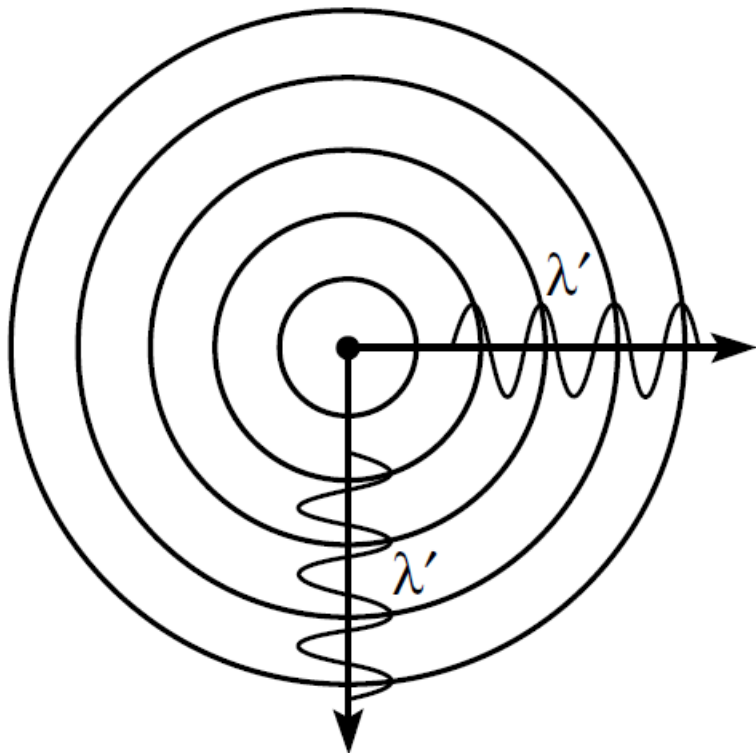


Laborsystem

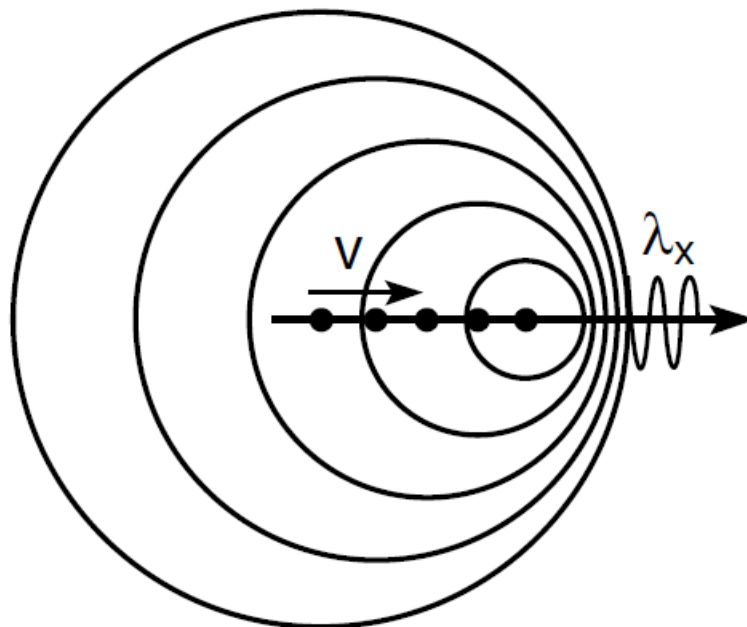


Synchrotron Radiation from Relativistic Electrons

$v \ll c$



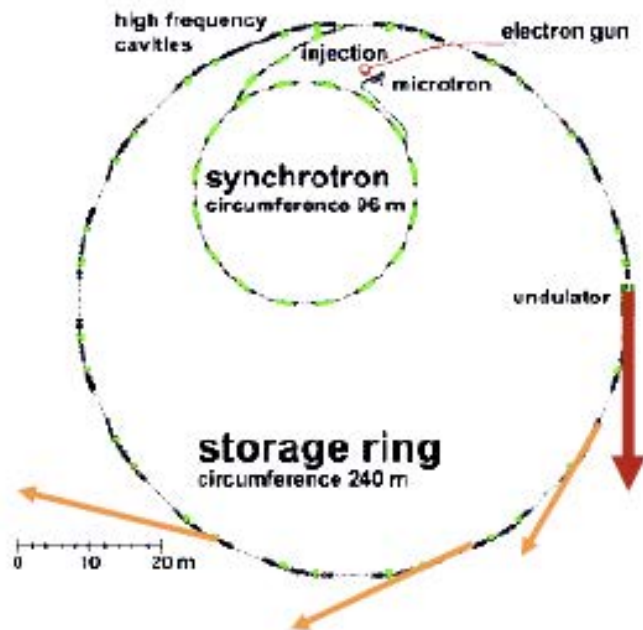
$v \lesssim c$

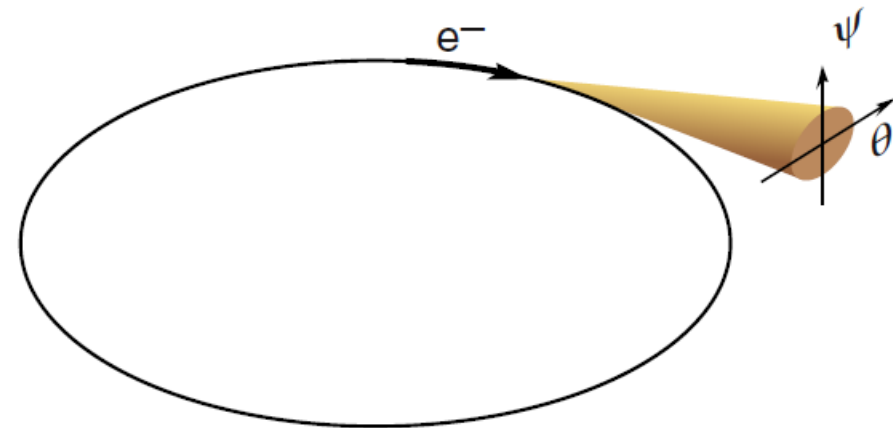


Note: Angle-dependent doppler shift

Synchrotronstrahlung

Abstrahlung im Ablenkmagnet





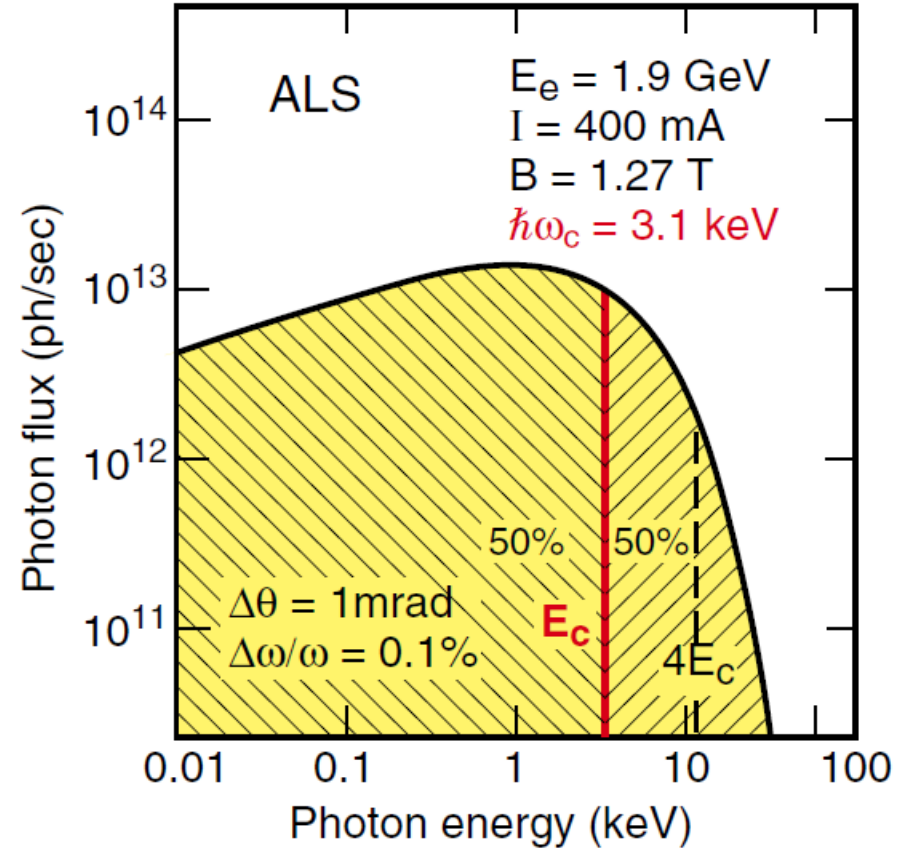
$$E_c = \hbar\omega_c = \frac{3e\hbar B\gamma^2}{2m} \quad (5.7a)$$

$$E_c(\text{keV}) = 0.6650 E_e^2(\text{GeV}) B(\text{T}) \quad (5.7b)$$

$$\gamma = \frac{E_e}{mc^2} = 1957 E_e(\text{GeV}) \quad (5.5)$$

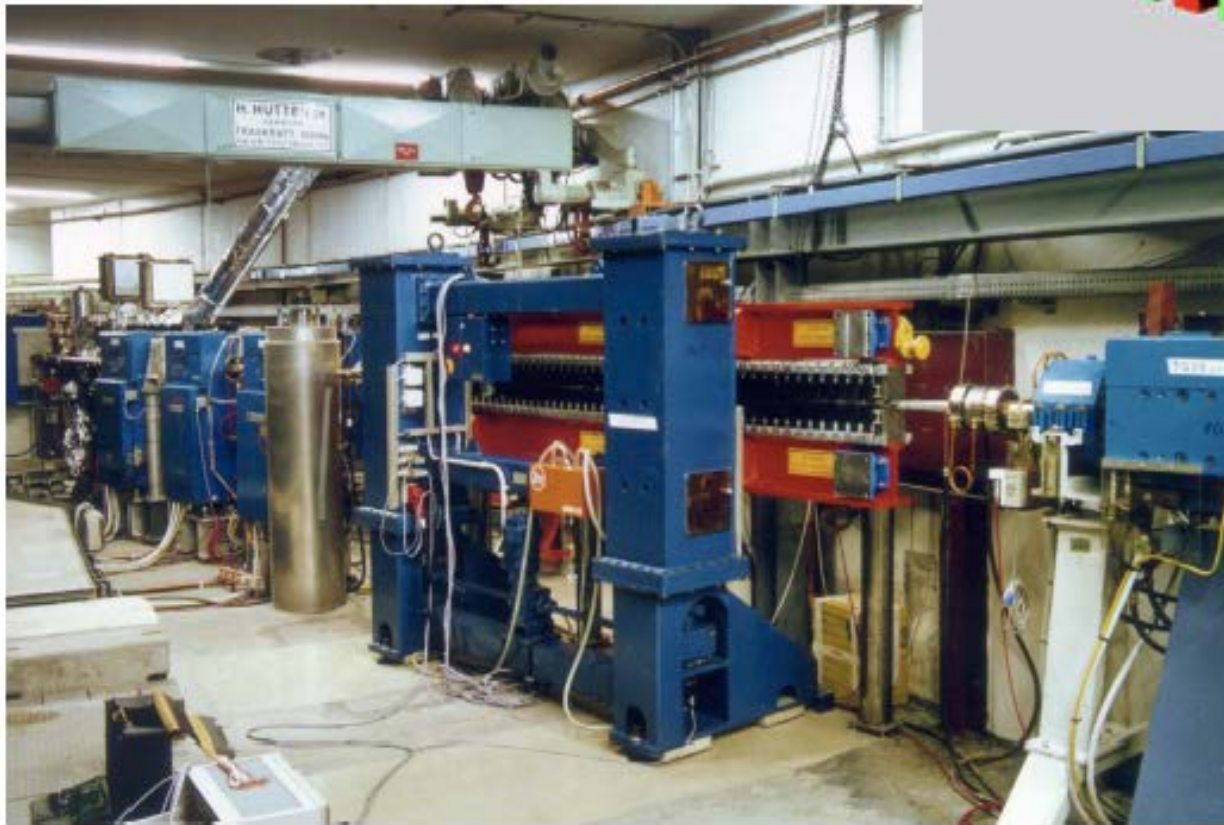
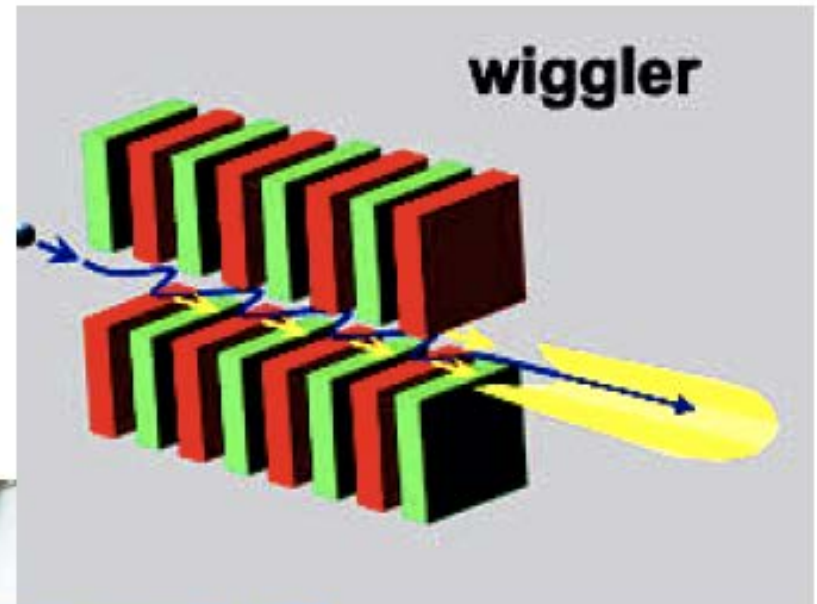
- Advantages:
- covers broad spectral range
 - least expensive
 - most accessible

- Disadvantages:
- limited coverage of hard x-rays
 - not as bright as undulator

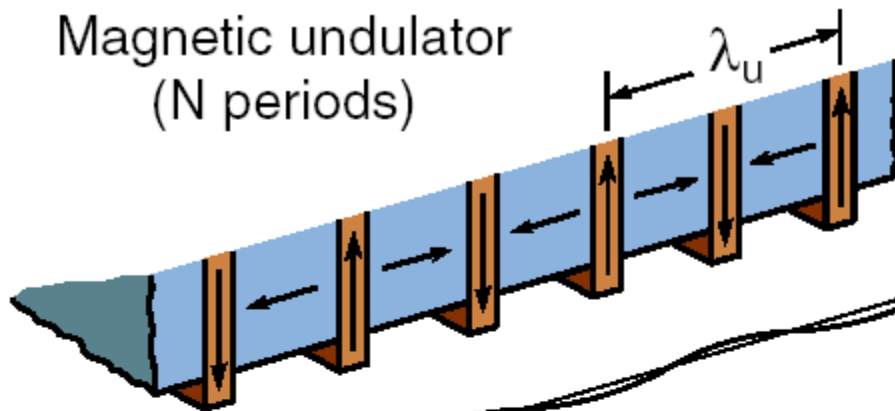


Synchrotronstrahlung

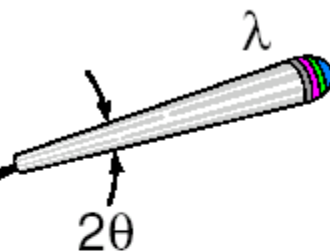
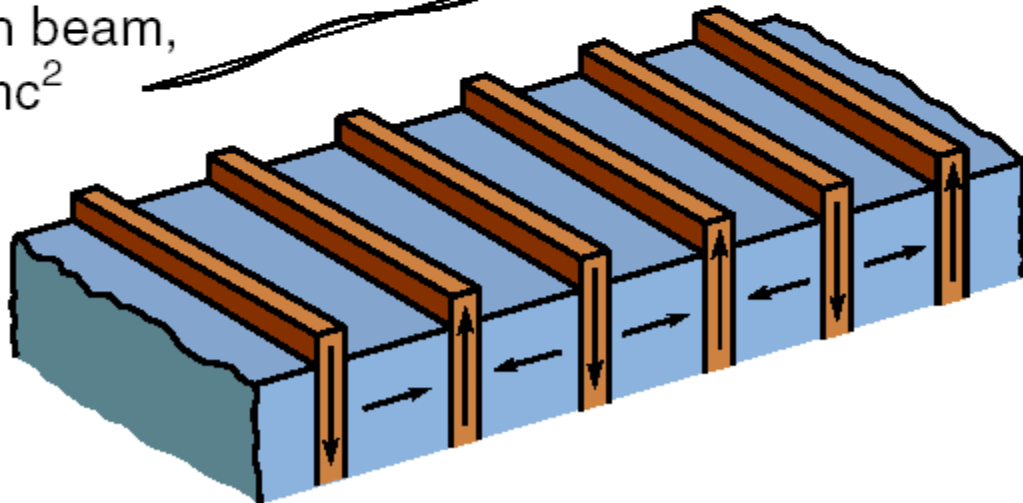
durch alternierende Magnete:
Überlagerung der Abstrahlprozesse



Magnetic undulator
(N periods)



Relativistic
electron beam,
 $E_e = \gamma mc^2$



$$\lambda \approx \frac{\lambda_u}{2\gamma^2}$$

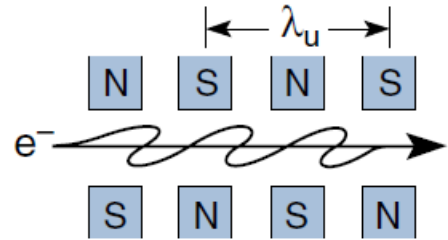
$$\theta_{\text{cen}} \approx \frac{1}{\gamma^* \sqrt{N}}$$

$$\left[\frac{\Delta\lambda}{\lambda} \right]_{\text{cen}} = \frac{1}{N}$$



Undulator Radiation

Laboratory Frame of Reference

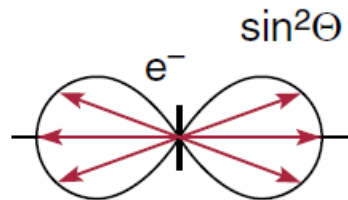


$$E = \gamma mc^2$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$N = \#$ periods

Frame of Moving e^-



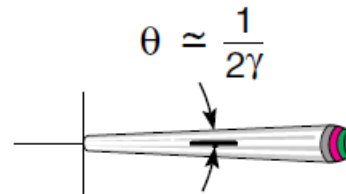
e^- radiates at the Lorentz contracted wavelength:

$$\lambda' = \frac{\lambda_u}{\gamma}$$

Bandwidth:

$$\frac{\lambda'}{\Delta\lambda'} \approx N$$

Frame of Observer



Doppler shortened wavelength on axis:

$$\lambda = \lambda' \gamma (1 - \beta \cos \theta)$$

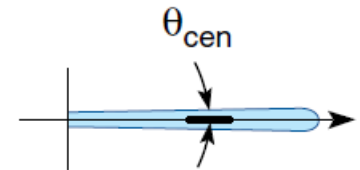
$$\lambda = \frac{\lambda_u}{2\gamma^2} (1 + \gamma^2 \theta^2)$$

Accounting for transverse motion due to the periodic magnetic field:

$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

where $K = eB_0\lambda_u/2\pi mc$

Following Monochromator



$$\text{For } \frac{\Delta\lambda}{\lambda} \approx \frac{1}{N}$$

$$\theta_{\text{cen}} \approx \frac{1}{\gamma\sqrt{N}}$$

typically

$$\theta_{\text{cen}} \approx 40 \mu\text{rad}$$

**Info über
Synchrotronstrahlung
im Internet**

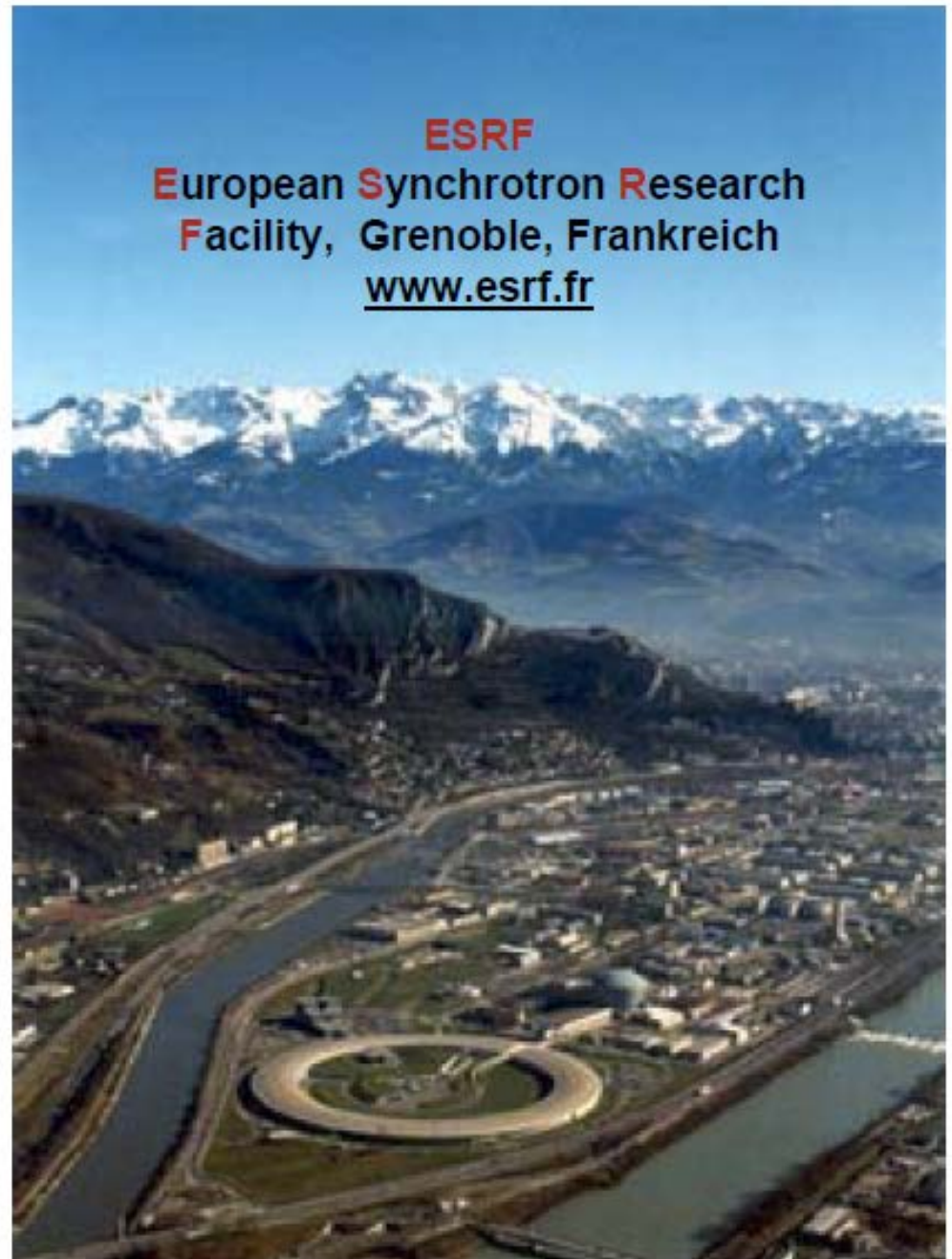
DESY

Deutsches Elektronensynchrotron
HASYLAB: Hamburger
Synchrotronstrahlungslabor
www-hasylab.desy.de

BESSY

Berliner Elektronenspeicherring –
Gesellschaft für Synchrotronstrahlung
www.bessy.de/guided_tour/

Preis ≥ € 1.000.000.000

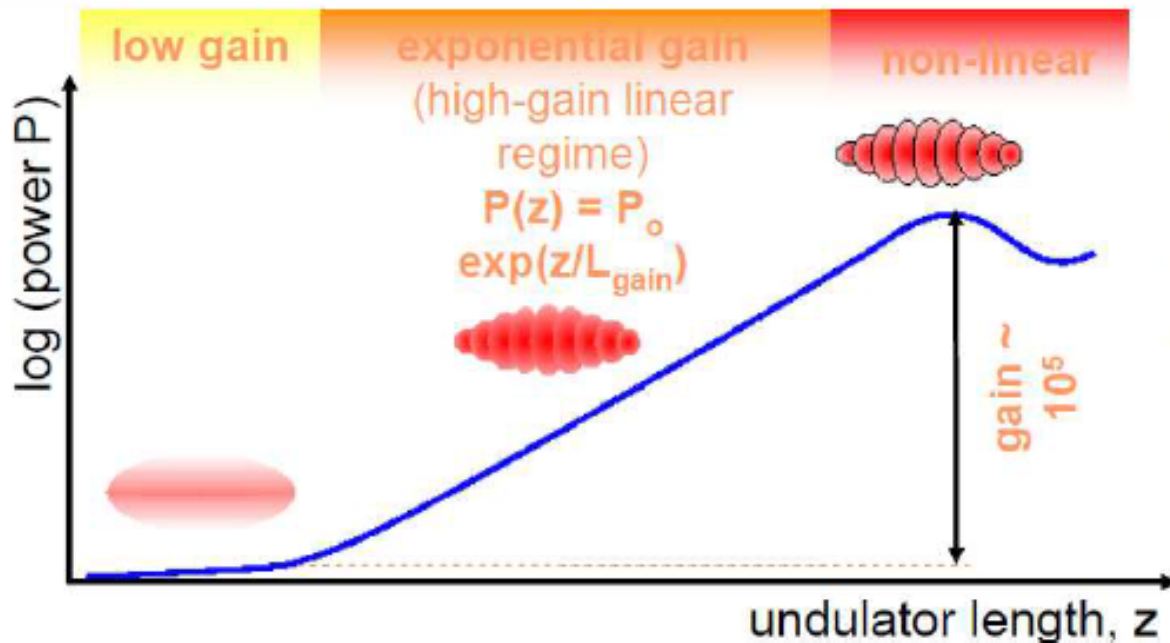
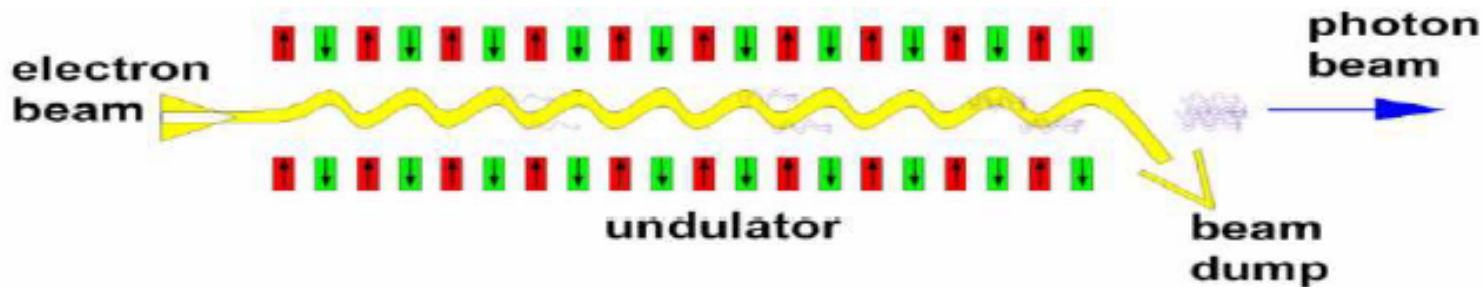


Höchstbrillante Röntgenquelle : Der Freie Elektronenlaser

SASE Prinzip

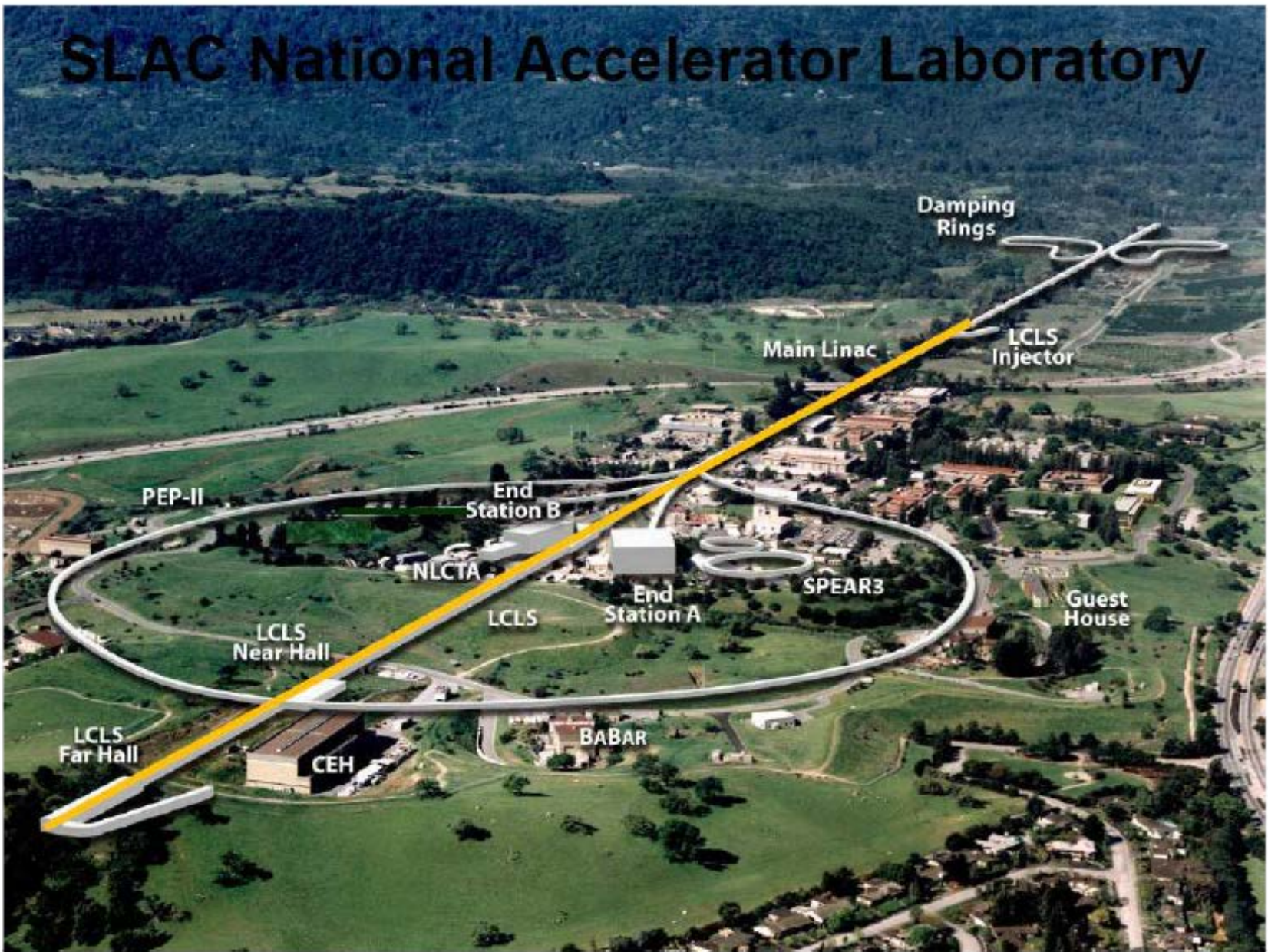
SASE = Self Amplification of Spontaneous Emission

Saldin, Schneidmiller, Yurkov



- Bunch wechselwirkt mit eigenem Photonfeld
 - Mikrobunche entstehen
 - Elektronen strahlen kohärent $\sim N_e^2$ mit $N_e \approx 10^6$
- **Verstärkung bis zu N_e !**

SLAC National Accelerator Laboratory



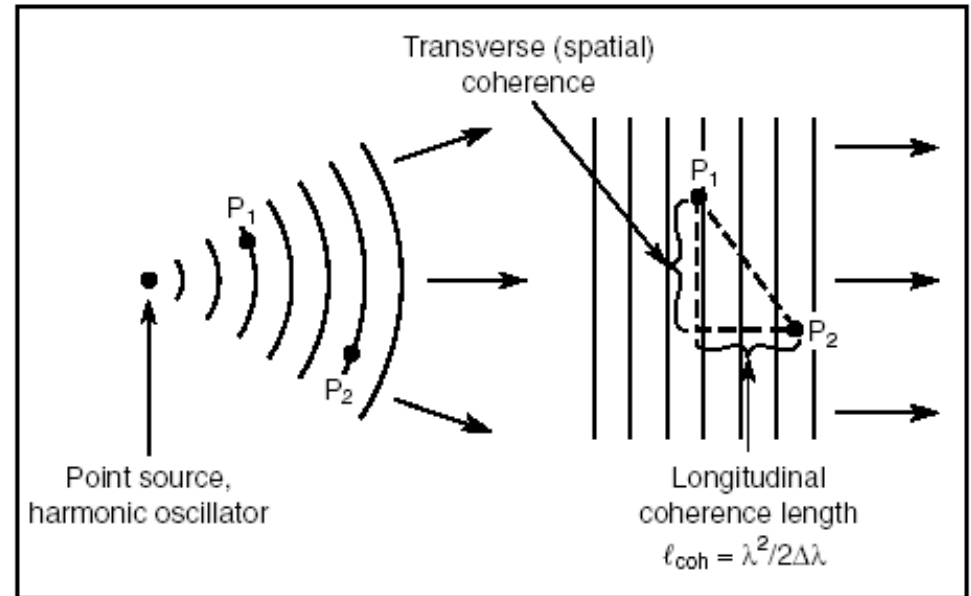
Mutual coherence factor

$$\Gamma_{12}(\tau) \equiv \langle E_1(t + \tau)E_2^*(t) \rangle \quad (8.1)$$

Normalize degree of spatial coherence
(complex coherence factor)

$$\mu_{12} = \frac{\langle E_1(t)E_2^*(t) \rangle}{\sqrt{\langle |E_1|^2 \rangle} \sqrt{\langle |E_2|^2 \rangle}} \quad (8.12)$$

A high degree of coherence ($\mu \rightarrow 1$) implies an ability to form a high contrast interference (fringe) pattern. A low degree of coherence ($\mu \rightarrow 0$) implies an absence of interference, except with great care. In general radiation is partially coherent.

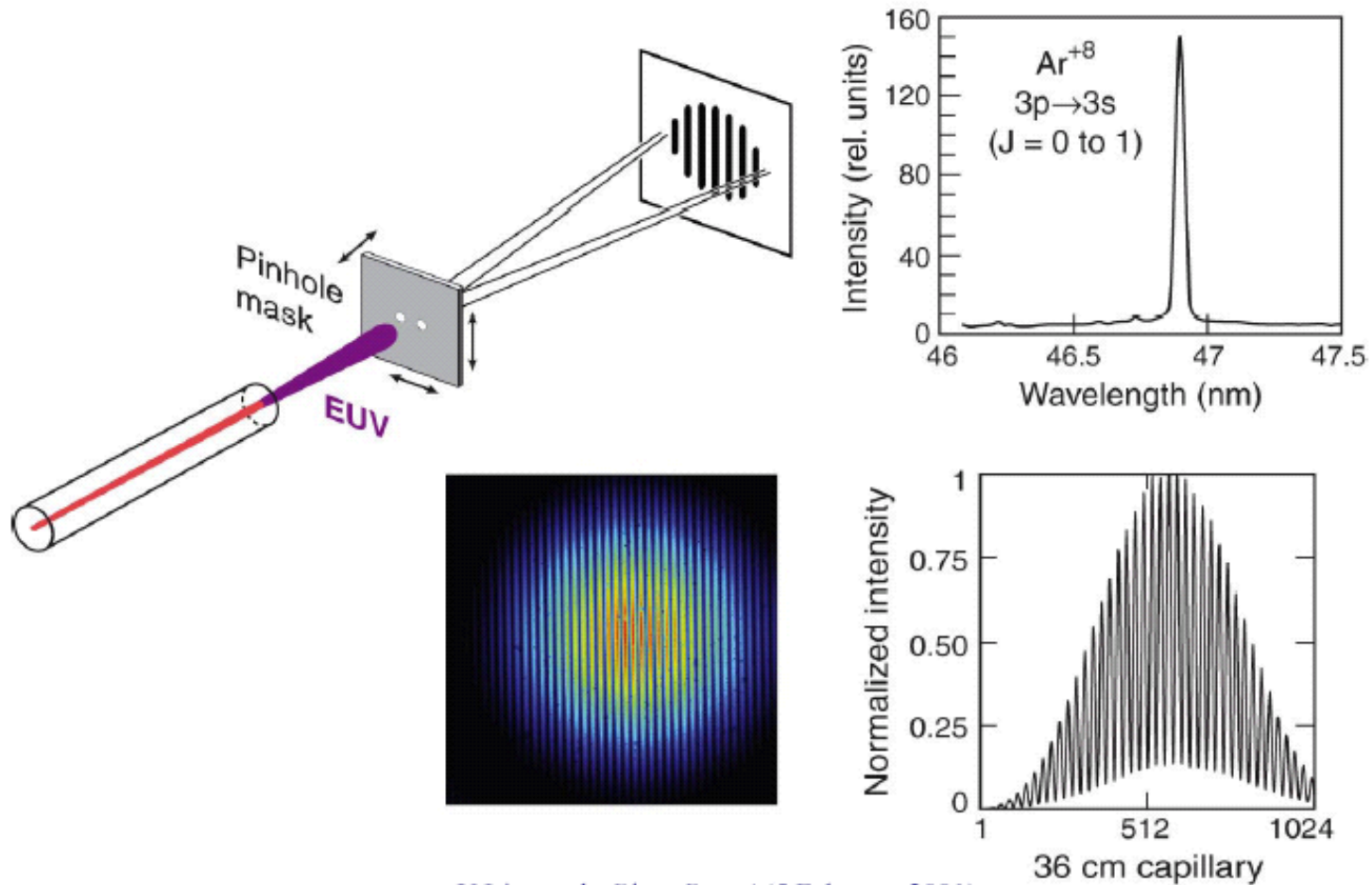


Longitudinal (temporal) coherence length

$$\ell_{coh} = \frac{\lambda^2}{2 \Delta\lambda} \quad (8.3)$$

Full spatial (transverse) coherence

$$d \cdot \theta = \lambda / 2\pi \quad (8.5)$$

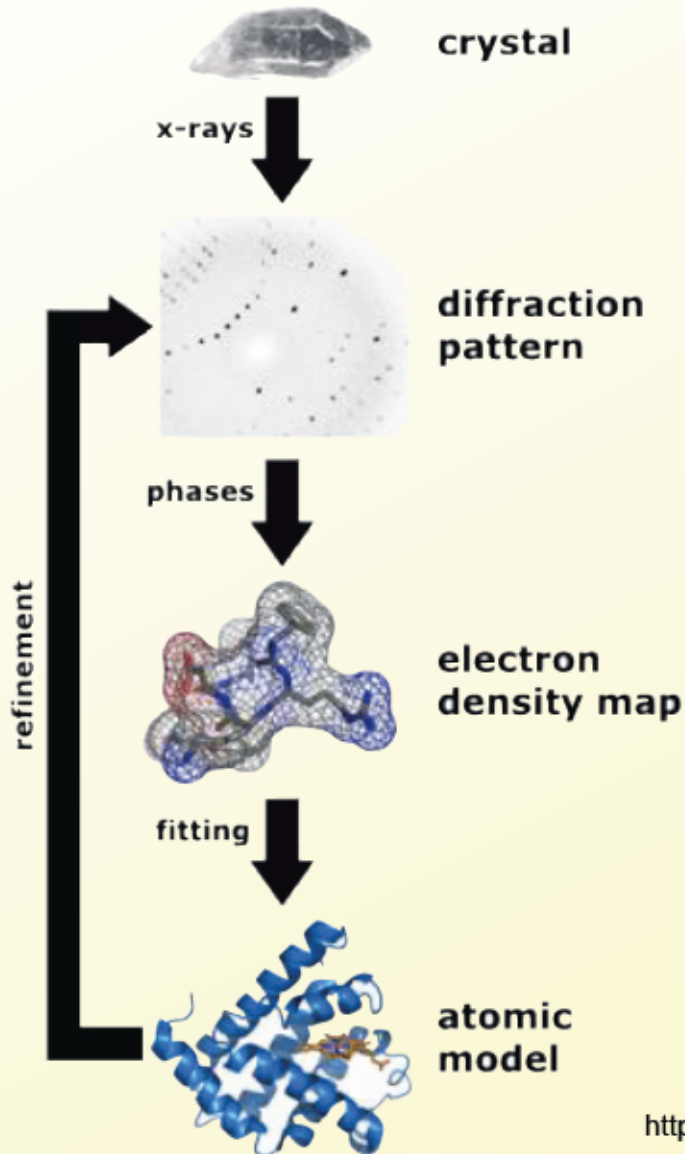


Y.Liu, et al., *Phys. Rev. A* (5 February 2001)

$$P_{\text{coh}} = 3 \text{ mW} \rightarrow 7 \times 10^{14} \text{ ph/sec @ } 46.9 \text{ nm}$$

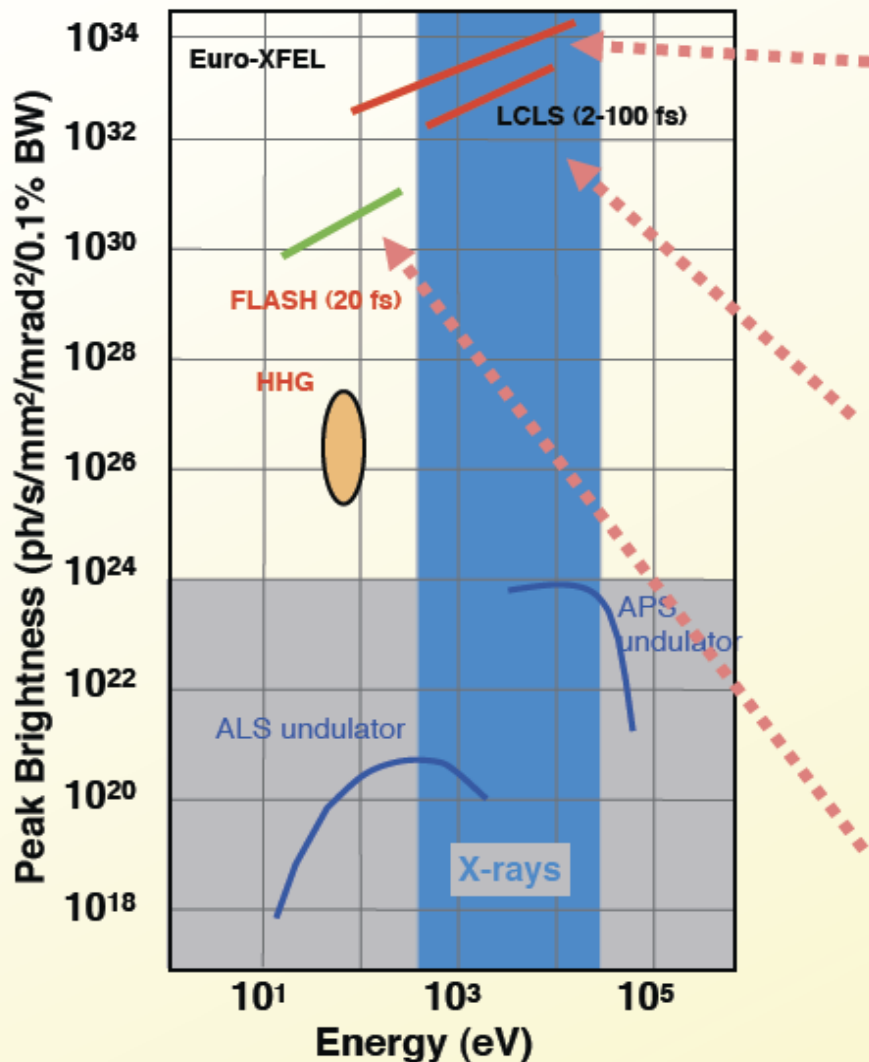
Courtesy of Prof. Jorge Rocca, Colorado State Univ.

Crystallography overcomes radiation damage and optics limitations, but requires crystals



- ★ Radiation damage is spread out over 10^{10} identical unit cells
- ★ Diffraction from unit cells adds up coherently to form strong Bragg peaks
- ★ ~ 60,000 structures solved (in protein data bank), but ~15,000 distinct structures
- The bottleneck is in growing crystals

X-ray free-electron lasers provide pulses that are *intense, short duration, short wavelength, and coherent*



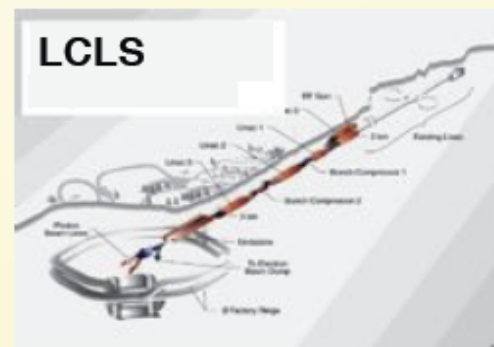
APS=Advanced Photon Source (ANL)
ALS=Advanced Light Source (LBNL)



operational 2013

12 keV, 50 fs, 10¹³ photons

European X-ray FEL,
DESY, Hamburg



LCLS

operational now

800 eV to 2 keV in 2009
8 keV, 100 fs, 10¹² photons

Linac Coherent Light Source,
SLAC, Stanford



FLASH

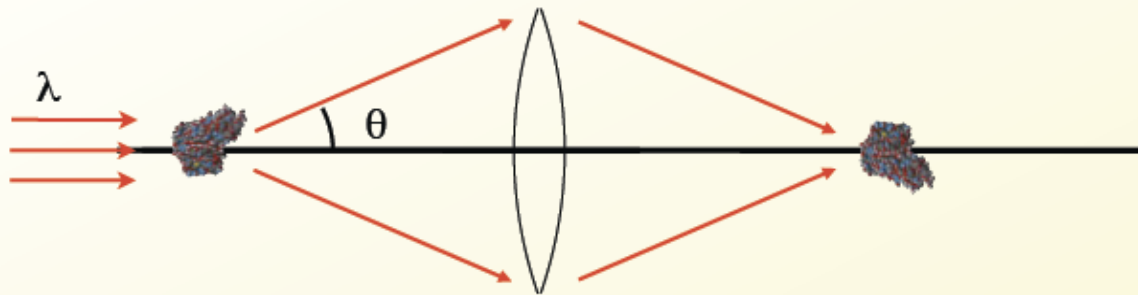
operational now

200 eV, 25 fs, 10¹² photons
upgrade to 300 eV, 400 fs

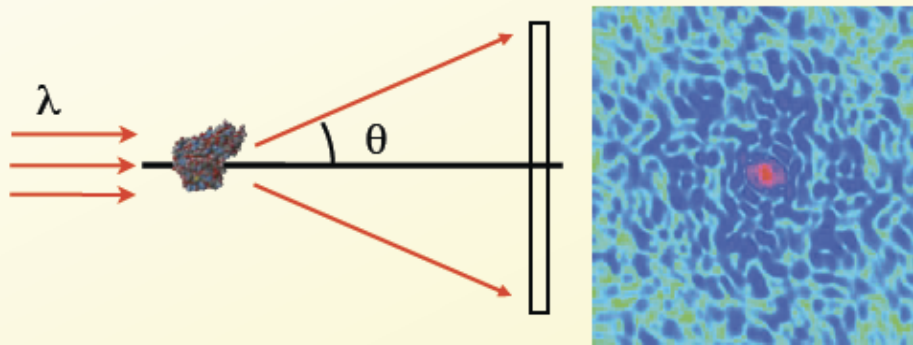
FLASH
DESY, Hamburg

Coherent diffractive imaging is lensless

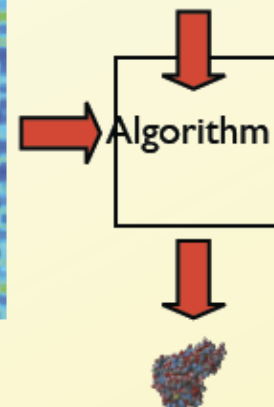
Use a computer to phase the scattered light, rather than a lens



A lens recombines the scattered rays with correct phases to give the image



Prior knowledge about object



An algorithm finds the phases that are consistent with measurements and prior knowledge

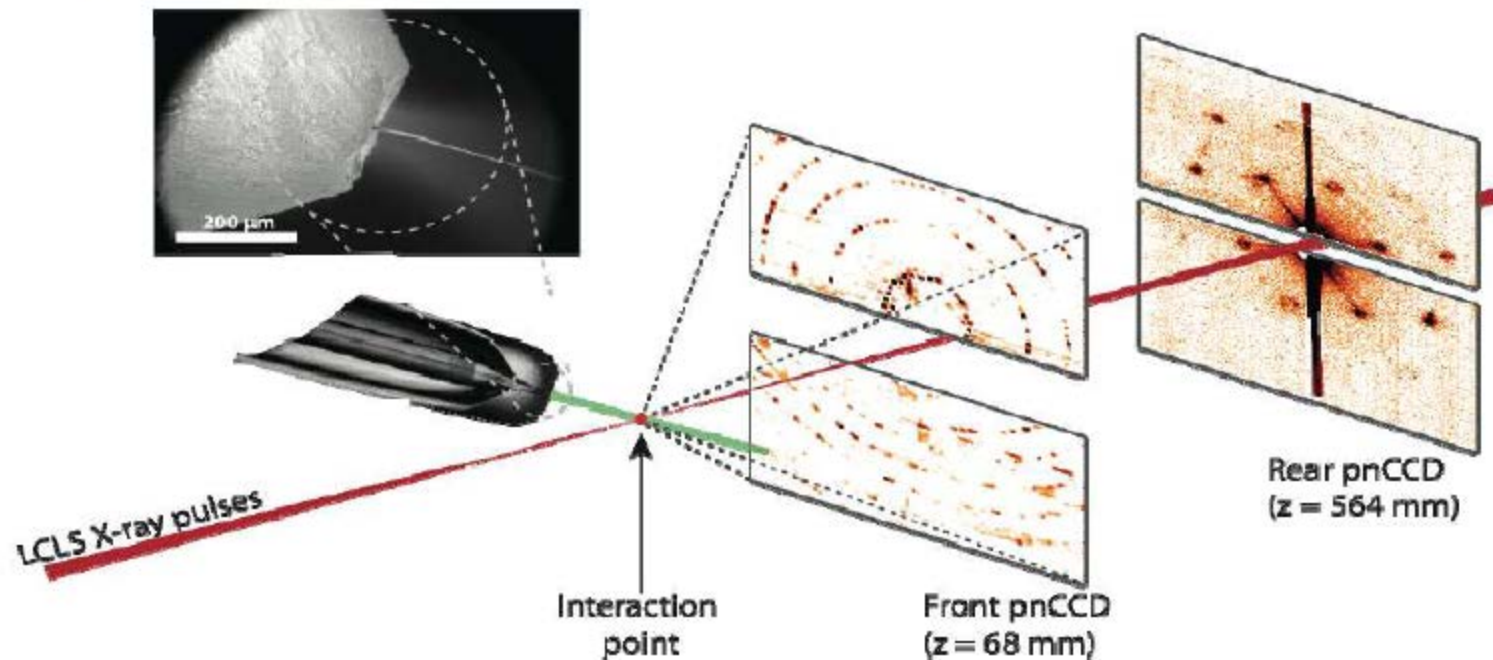
Resolution: $\delta = \lambda / \sin \theta$

J. Fienup, *Appl. Opt.* **21** 2758 (1987)

J. Miao et al, *Nature* **400** 342 (1999)

Nanocrystallography carried out in a flowing water microjet

- Single pulse diffraction from Photosystem 1 nanocrystals at LCLS
- $E = 1.8 \text{ keV}$
- $<10, 60, 200 \text{ fs}$ pulse
- 2 mJ pulse energy
- patterns collected at 30 Hz
- hit rate $>50\%$
- 5 Tb data in one night!



Chapter One :

- **Basic Absorption and Emission Processes**
- **Atomic Energy Levels and Allowed Transitions**
- **Scattering, Diffraction and Refraction**



Photon Energy, Wavelength, Power

$$\hbar\omega \cdot \lambda = hc = 1239.842 \text{ eV nm} \quad (1.1)$$

$$1 \text{ joule} \Rightarrow 5.034 \times 10^{15} \lambda[\text{nm}] \text{ photons} \quad (1.2a)$$

$$1 \text{ watt} \Rightarrow 5.034 \times 10^{15} \lambda[\text{nm}] \frac{\text{photons}}{\text{s}} \quad (1.2b)$$



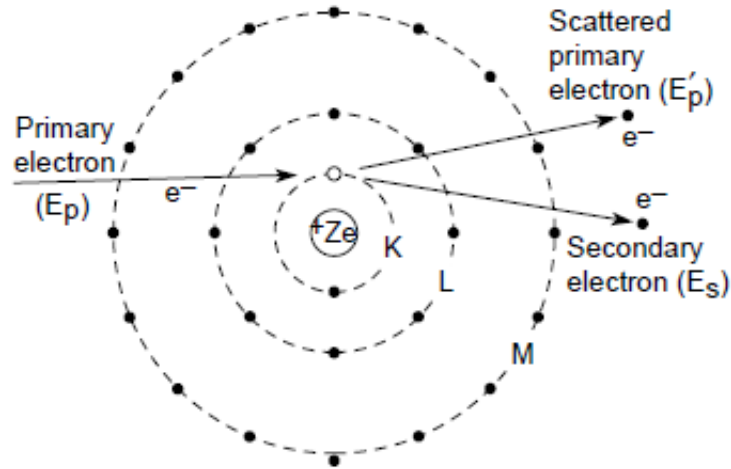
K and L₃-Absorption Edges for Selected Elements

Element	Z	K _{abs} -edge (eV)	L _{abs} -edge (eV)	λ _{K-abs} (nm)	λ _{L-abs} (nm)	I _{abs}	
						100 eV (nm)	1 keV (μm)
Be	4	112	—	11.1	—	730	9.0
C	6	284	—	4.36	—	190	2.1
N	7	410	—	3.02	—	—	—
O	8	543	—	2.28	—	—	—
H ₂ O						160	2.3
Al	13	1,560	73	0.795	17.1	34	3.1
Si	14	1,839	99	0.674	12.5	63	2.7
S	16	2,472	163	0.502	7.63	330	1.9
Ca	20	4,039	346	0.307	3.58	290	1.3
Ti	22	4,966	454	0.250	2.73	65	0.38
V	23	5,465	512	0.227	2.42	46	0.26
Cr	24	5,989	574	0.207	2.16	31	0.19
Fe	26	7,112	707	0.174	1.75	22	0.14
Ni	28	8,333	853	0.149	1.45	16	0.11
Cu	29	8,979	933	0.138	1.33	18	0.10
Se	34	12,658	1,434	0.0979	0.865	63	0.96
Mo	42	20,000	2,520	0.0620	0.492	200	0.19
Sn	50	29,200	3,929	0.0425	0.316	17	0.17
Xe	54	34,561	4,782	0.0359	0.259	—	—
W	74	69,525	10,207	0.0178	0.121	28	0.13
Au	79	80,725	11,919	0.0154	0.104	28	0.10

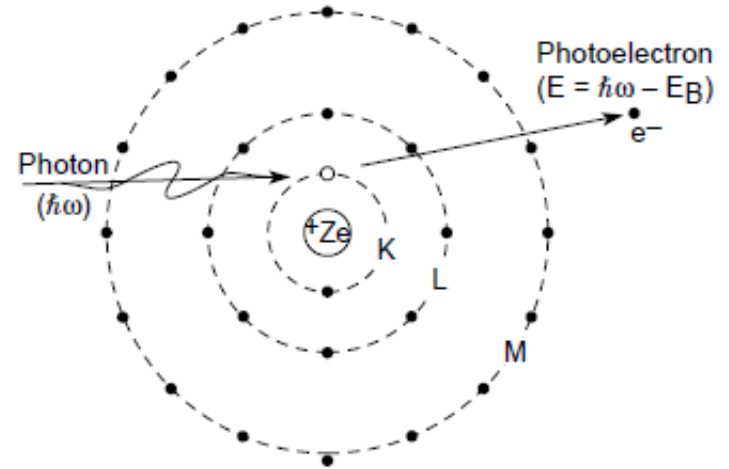


Basic Ionization and Emission Processes in Isolated Atoms

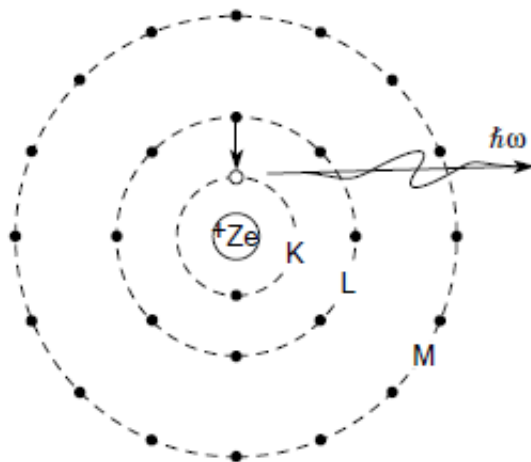
(a) Electron collision induced ionization



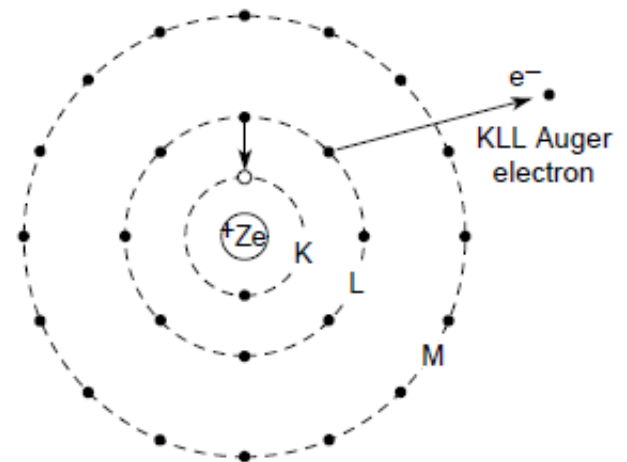
(b) Photoionization



(c) Fluorescent emission of characteristic radiation

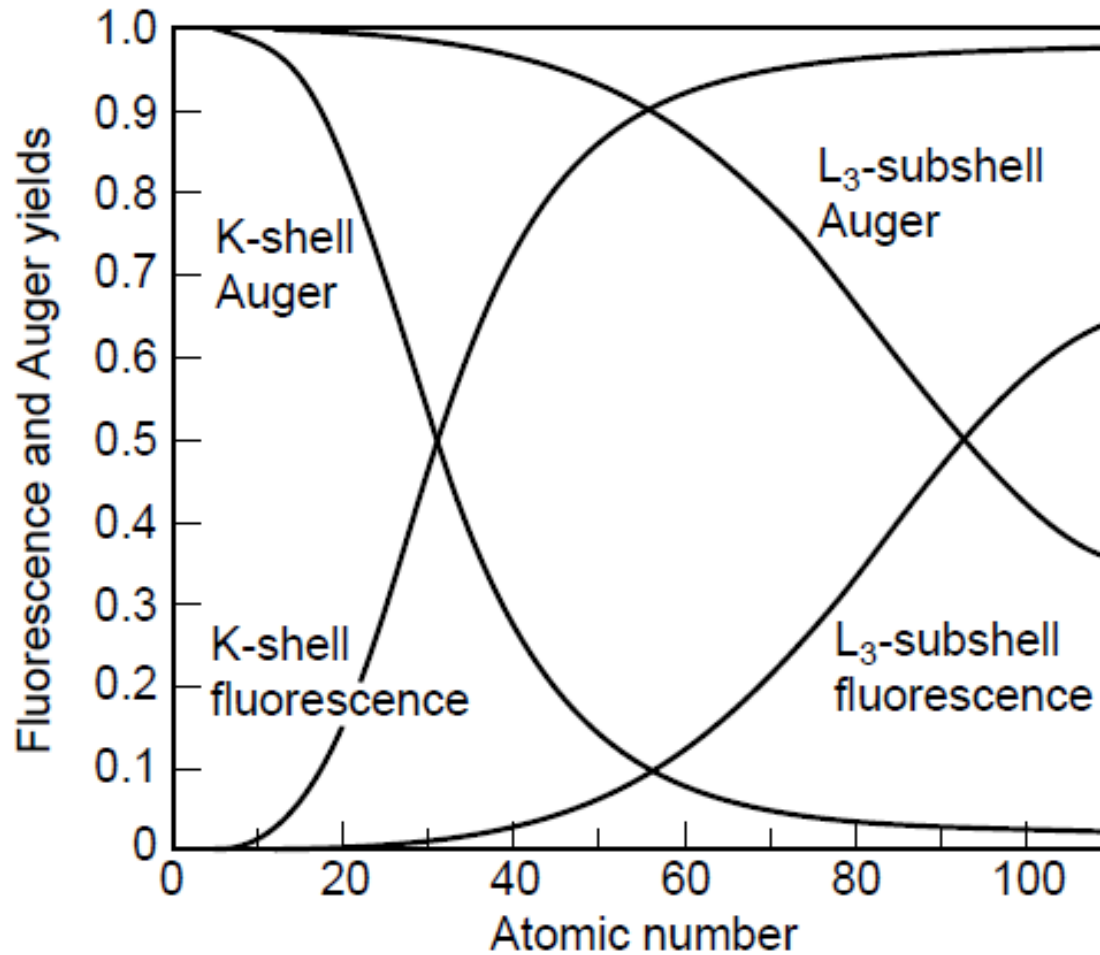


(d) Non-radiative Auger process





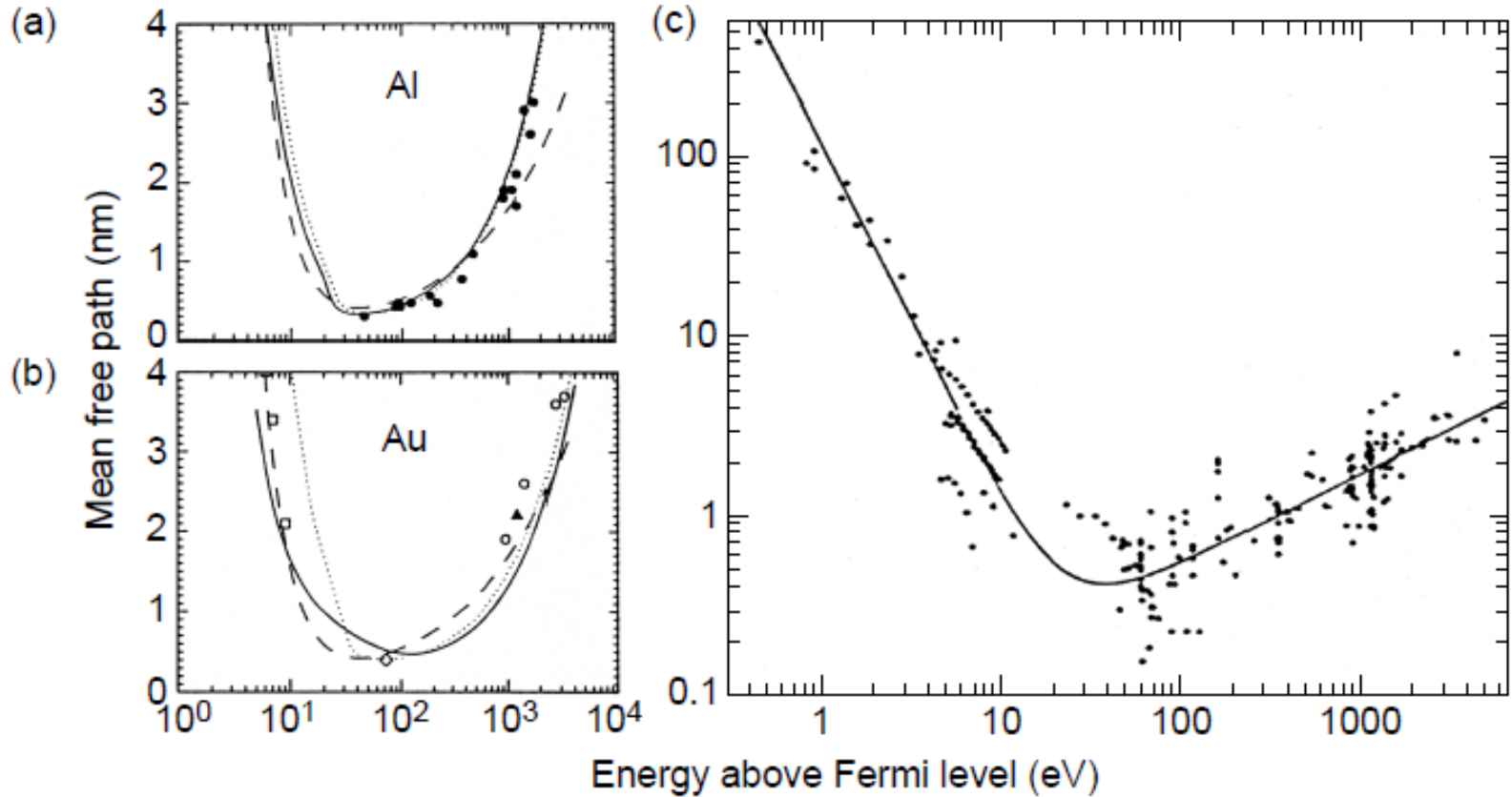
Fluorescence and Auger Emission Yields



(Courtesy of M. Krause, Oak Ridge)



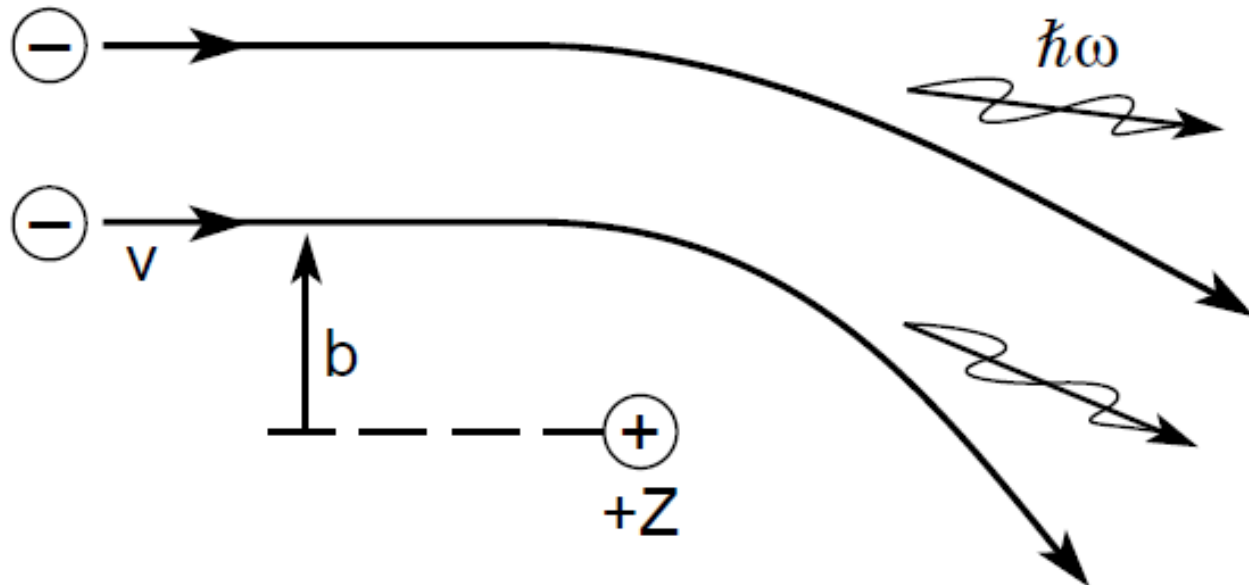
Electron Mean Free Paths As a Function of Energy



Courtesy of: Penn (a & b), Seah and Dench (c)

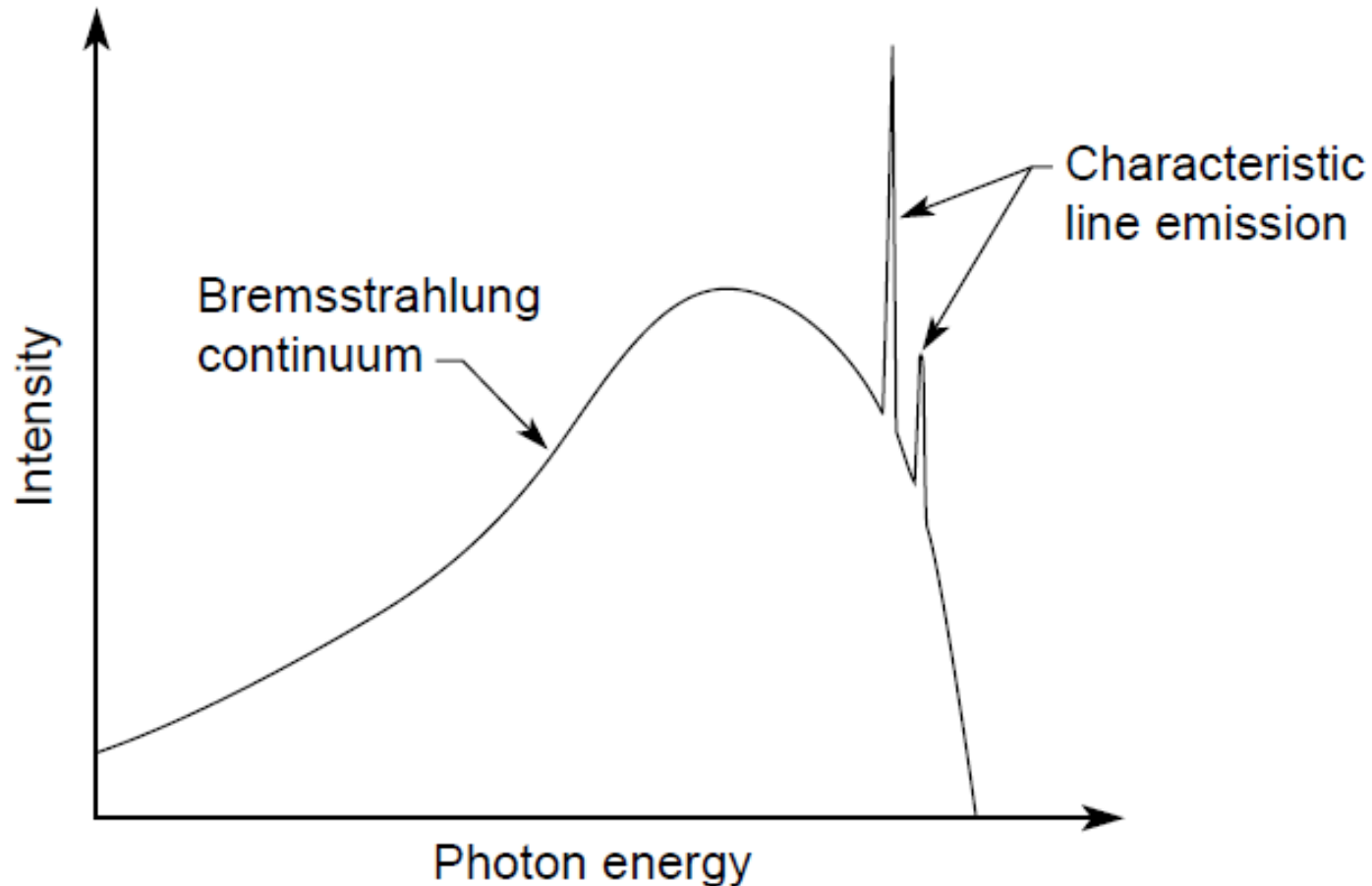


Bremsstrahlung Radiation: Braking Radiation from an Accelerated Charge



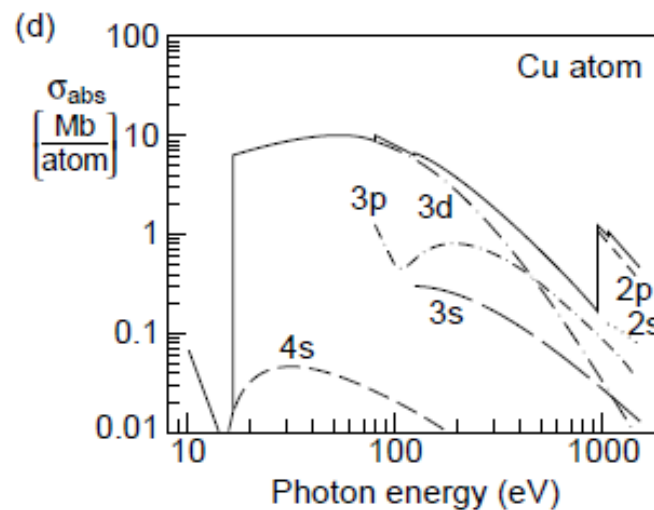
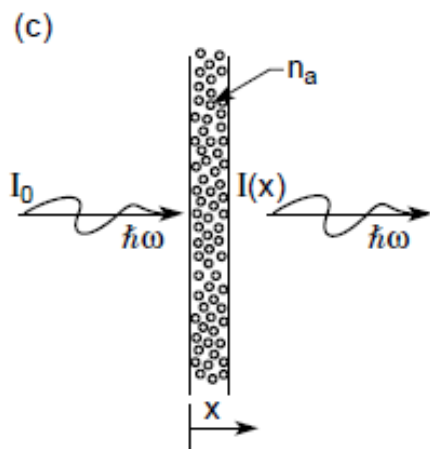
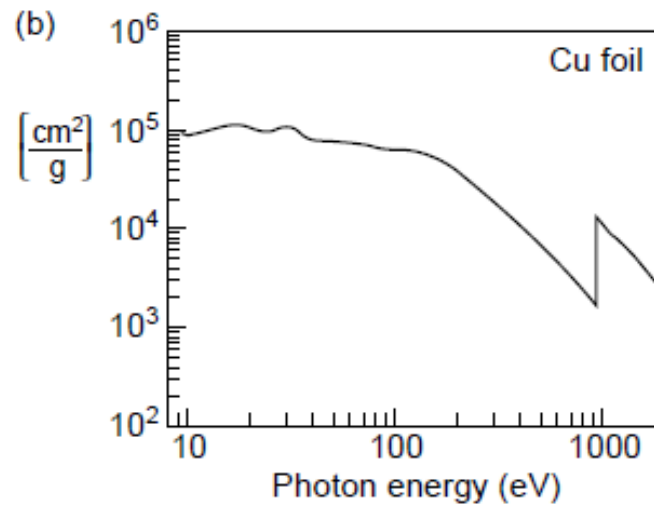
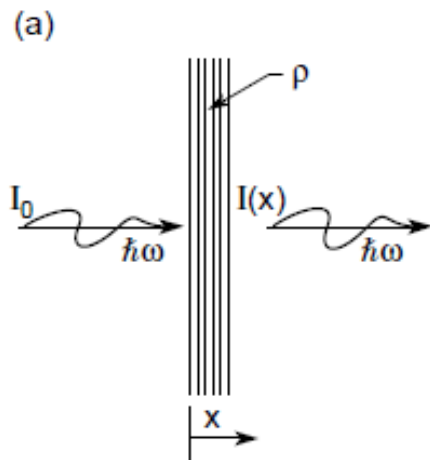


Continuum Bremsstrahlung Radiation and Narrow Characteristic Line Emission from a Solid Target with Electron Bombardment

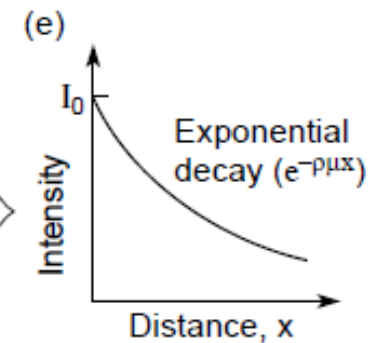




Photoabsorption by Thin Foils and Isolated Atoms



$$\frac{I}{I_0} = e^{-\rho\mu x}$$



$$\frac{I}{I_0} = e^{-n_a\sigma_{\text{abs}}x}$$



Atomic Energy Levels and Allowed Transitions in the Bohr Atom

Equate Coulomb Force $Ze^2/4\pi\epsilon_0r^2$ to the centripetal force mv^2/r :

$$E_n = \frac{mZ^2e^4}{32\pi^2\epsilon_0^2\hbar^2} \frac{1}{n^2} \quad (1.4)$$

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2Z} \cdot n^2 \quad (1.5)$$

$$\hbar\omega = E_i - E_f = \underbrace{\frac{me^4}{32\pi^2\epsilon_0\hbar^2}}_{13.6 \text{ eV}} \left[\frac{1}{n_f^2} - \frac{1}{n_i^2} \right] Z^2 \quad (1.6)$$

$$r_n = \frac{a_0n^2}{Z} \quad ; \quad a_0 = 0.529 \text{ \AA} \quad (1.9)$$



Quantum Mechanics Based on a Probabilistic Wave Function, $\Psi(\mathbf{r}, t)$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}, t) \Psi(\mathbf{r}, t) = i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} \quad (1.10)$$

$$P(\mathbf{r}, t) d\mathbf{r} = \Psi^*(\mathbf{r}, t) \Psi(\mathbf{r}, t) d\mathbf{r} \quad (1.13)$$

$$\bar{\mathbf{r}} = \iiint \mathbf{r} P(\mathbf{r}, t) d\mathbf{r} = \iiint \Psi^*(\mathbf{r}, t) \mathbf{r} \Psi(\mathbf{r}, t) d\mathbf{r} \quad (1.15)$$

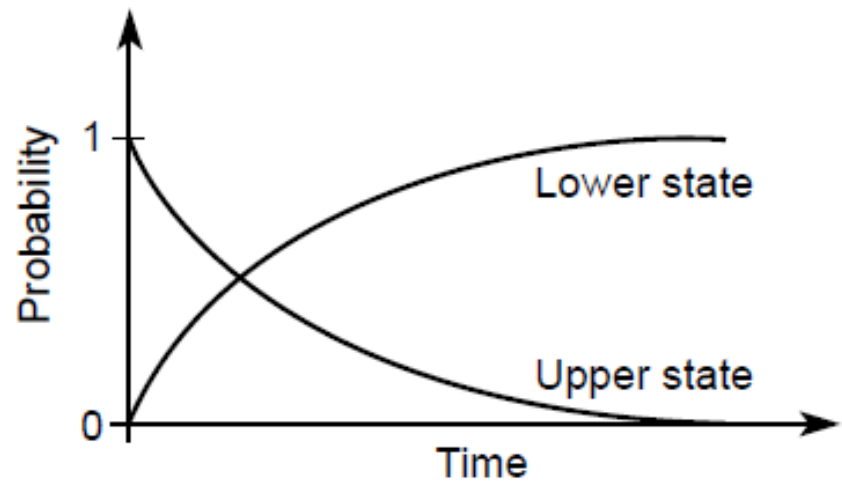
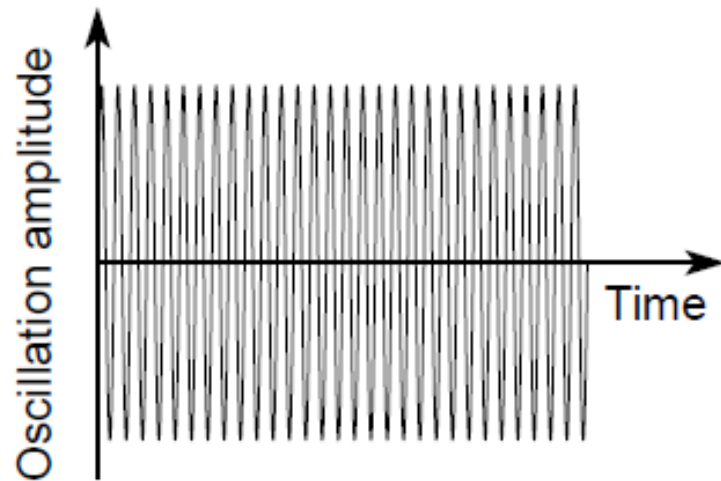
quantum numbers: n, ℓ, m_ℓ, m_s

selection rules for allowed transitions: $\Delta \ell = \pm 1$

$$\Delta j = 0, \pm 1$$

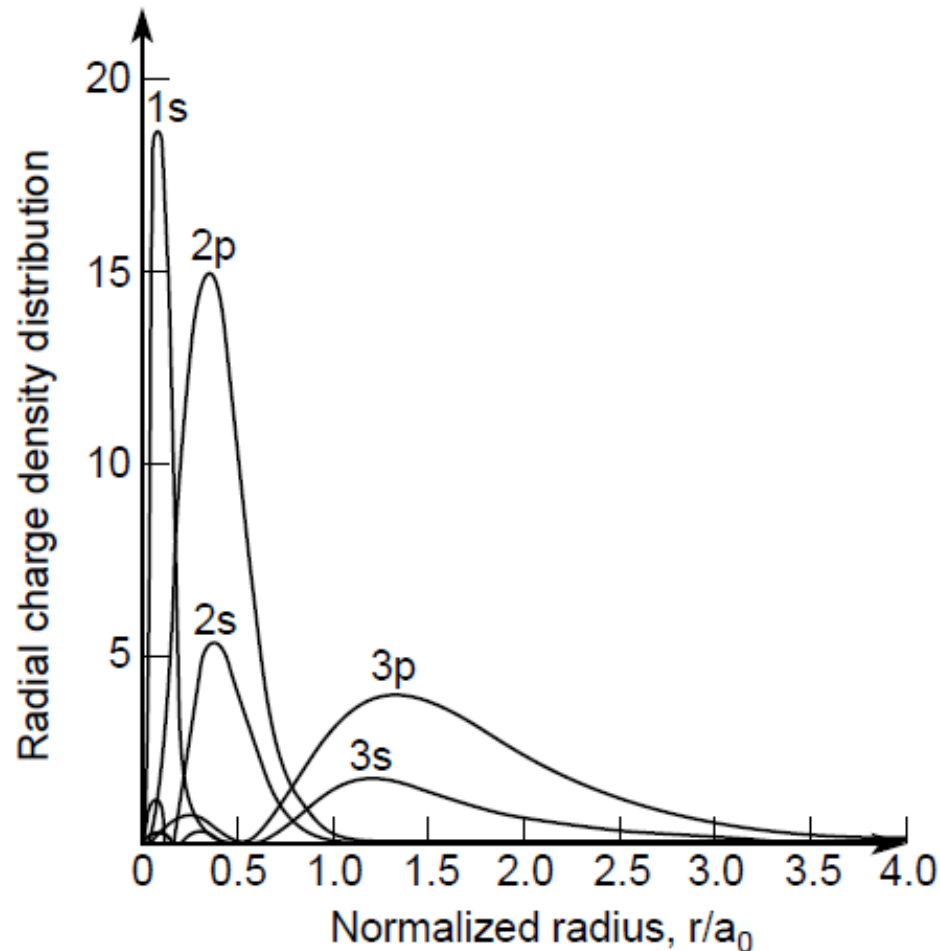


Radiative Decay Involves An Atom Oscillating Between Two Stationary States at the Frequency $\omega_{if} = (E_i - E_f) / \hbar$





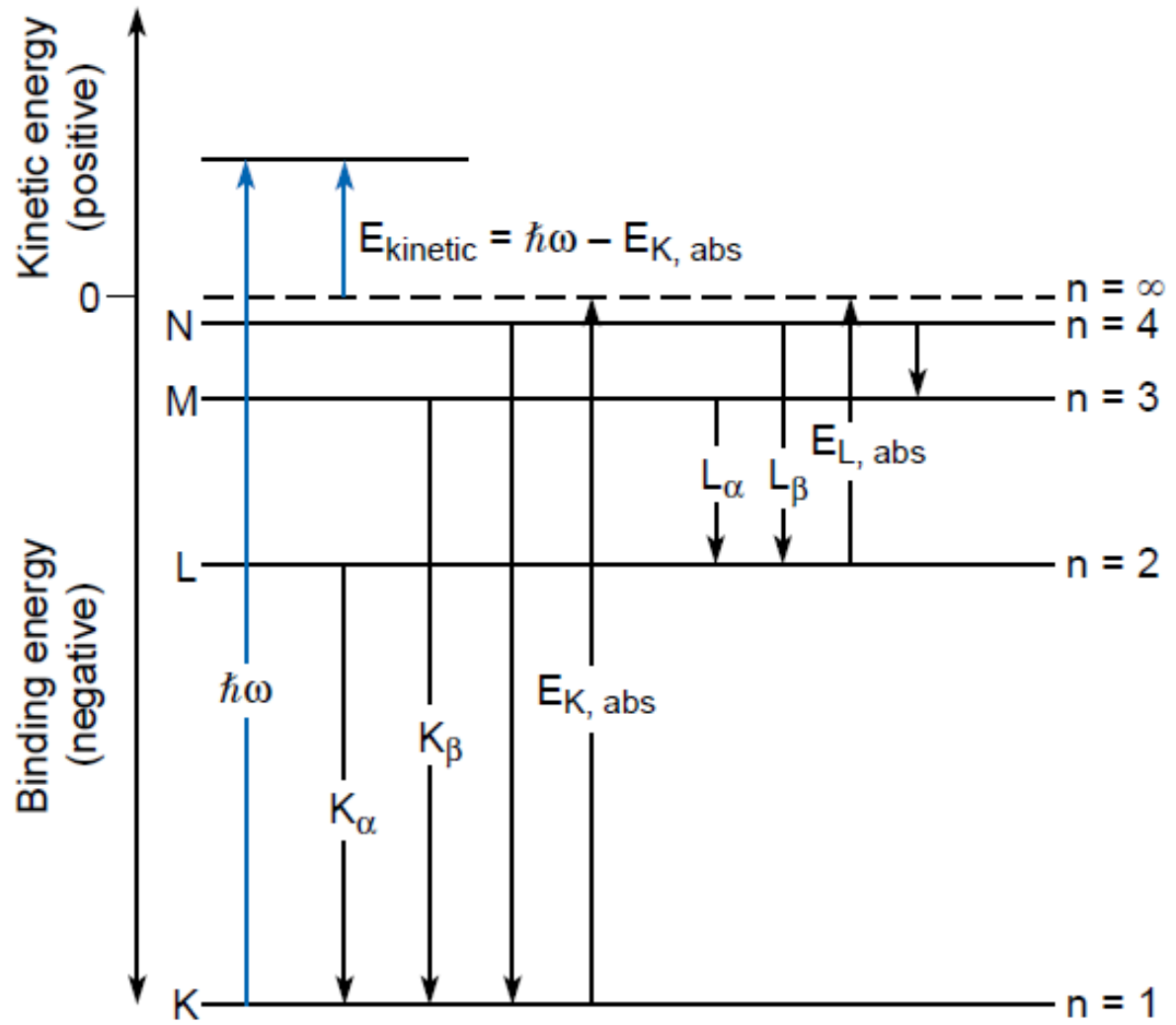
Probabilistic Radial Charge Distribution ($e/\text{\AA}$) in the Argon Atom



Courtesy of Eisberg and Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*.

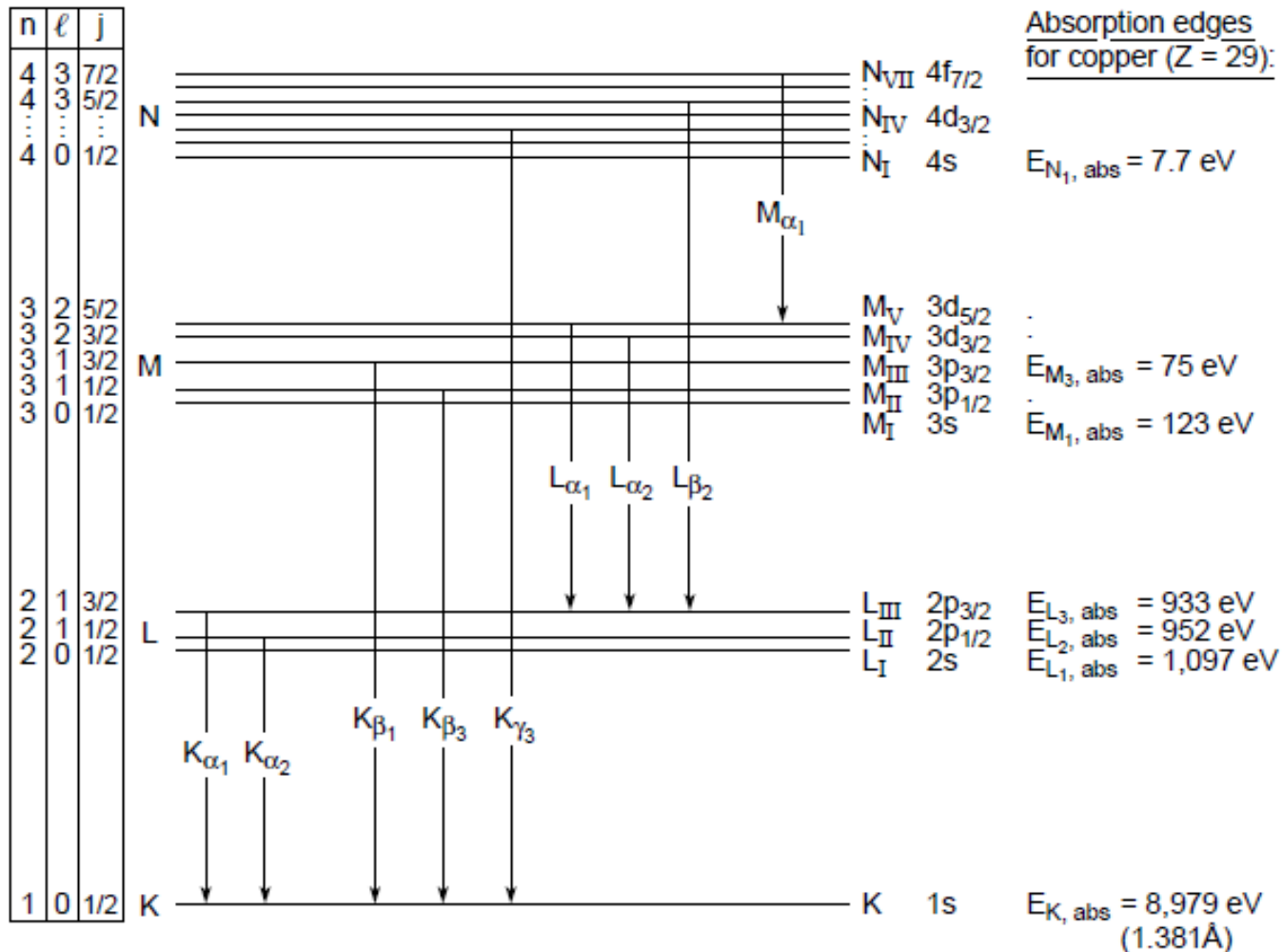


Energy Levels, Absorption Edges, and Characteristic Line Emissions for a Multi-Electron Atom





Energy Levels, Quantum Numbers, and Allowed Transitions for the Copper Atom



Cu K_{α₁} = 8,048 eV (1.541Å)

Cu K_{α₂} = 8,028 eV (1.544Å)

Cu K_{β₁} = 8,905 eV

Cu L_{α₁} = 930 eV

Cu L_{α₂} = 930 eV

Cu L_{β₁} = 950 eV



TABLE B.1. Electron binding energies in electron volts for the elements in their natural forms.^a

Element	K 1s	L ₁ 2s	L ₂ 2p _{1/2}	L ₃ 2p _{3/2}	M ₁ 3s	M ₂ 3p _{1/2}	M ₃ 3p _{3/2}	M ₄ 3d _{3/2}	M ₅ 3d _{5/2}	N ₁ 4s	N ₂ 4p _{1/2}	N ₃ 4p _{3/2}
1 H	13.6											
2 He	24.6 ^b											
3 Li	54.7 ^b											
4 Be	111.5 ^b											
5 B	188 ^b											
6 C	284.2 ^b											
7 N	409.9 ^b	37.3 ^b										
8 O	543.1 ^b	41.6 ^b										
9 F	696.7 ^b											
10 Ne	870.2 ^b	48.5 ^b	21.7 ^b	21.6 ^b								
11 Na	1070.8 ^c	63.5 ^c	30.4 ^c	30.5 ^b								
12 Mg	1303.0 ^c	88.6 ^b	49.6 ^c	49.2 ^c								
13 Al	1559.6	117.8 ^b	72.9 ^b	72.5 ^b								
14 Si	1838.9	149.7 ^b	99.8 ^b	99.2 ^b								
15 P	2145.5	189 ^b	136 ^b	135 ^b								
16 S	2472	230.9 ^b	163.6 ^b	162.5 ^b								
17 Cl	2822.4	270.2 ^b	202 ^b	200 ^b								
18 Ar	3205.9 ^b	326.3 ^b	250.6 ^b	248.4 ^b	29.3 ^b	15.9 ^b	15.7 ^b					
19 K	3608.4 ^b	378.6 ^b	297.3 ^b	294.6 ^b	34.8 ^b	18.3 ^b	18.3 ^b					
20 Ca	4038.5 ^b	438.4 ^c	349.7 ^c	346.2 ^c	44.3 ^c	25.4 ^c	25.4 ^c					
21 Sc	4492.8	498.0 ^b	403.6 ^b	398.7 ^b	51.1 ^b	28.3 ^b	28.3 ^b					
22 Ti	4966.4	560.9 ^c	461.2 ^c	453.8 ^c	58.7 ^c	32.6 ^c	32.6 ^c					
23 V	5465.1	626.7 ^c	519.8 ^c	512.1 ^c	66.3 ^c	37.2 ^c	37.2 ^c					
24 Cr	5989.2	695.7 ^c	583.8 ^c	574.1 ^c	74.1 ^c	42.2 ^c	42.2 ^c					
25 Mn	6539.0	769.1 ^c	649.9 ^c	638.7 ^c	82.3 ^c	47.2 ^c	47.2 ^c					
26 Fe	7112.0	844.6 ^c	719.9 ^c	706.8 ^c	91.3 ^c	52.7 ^c	52.7 ^c					
27 Co	7708.9	925.1 ^c	793.3 ^c	778.1 ^c	101.0 ^c	58.9 ^c	58.9 ^c					
28 Ni	8332.8	1008.6 ^c	870.0 ^c	852.7 ^c	110.8 ^c	68.0 ^c	66.2 ^c					
29 Cu	8978.9	1096.7 ^c	952.3 ^c	932.5 ^c	122.5 ^c	77.3 ^c	75.1 ^c					
30 Zn	9658.6	1196.2 ^b	1044.9 ^b	1021.8 ^b	139.8 ^b	91.4 ^b	88.6 ^b	10.2 ^b	10.1 ^b			

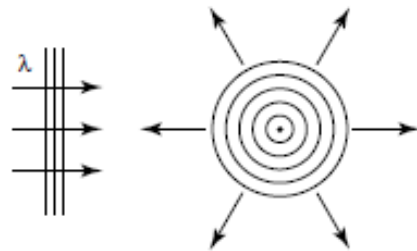
**TABLE B.2.** Photon energies, in electron volts, of principal K and L shell emission lines.^a

Element	K α_1	K α_2	K β_1	L α_1	L α_2	L β_1	L β_2	L γ_1
3 Li	54.3							
4 Be	108.5							
5 B	183.3							
6 C	277							
7 N	392.4							
8 O	524.9							
9 F	676.8							
10 Ne	848.6	848.6						
11 Na	1,040.98	1,040.98	1,071.1					
12 Mg	1,253.60	1,253.60	1,302.2					
13 Al	1,486.70	1,486.27	1,557.45					
14 Si	1,739.98	1,739.38	1,835.94					
15 P	2,013.7	2,012.7	2,139.1					
16 S	2,307.84	2,306.64	2,464.04					
17 Cl	2,622.39	2,620.78	2,815.6					
18 Ar	2,957.70	2,955.63	3,190.5					
19 K	3,313.8	3,311.1	3,589.6					
20 Ca	3,691.68	3,688.09	4,012.7	341.3	341.3	344.9		
21 Sc	4,090.6	4,086.1	4,460.5	395.4	395.4	399.6		
22 Ti	4,510.84	4,504.86	4,931.81	452.2	452.2	458.4		
23 V	4,952.20	4,944.64	5,427.29	511.3	511.3	519.2		
24 Cr	5,414.72	5,405.509	5,946.71	572.8	572.8	582.8		
25 Mn	5,898.75	5,887.65	6,490.45	637.4	637.4	648.8		
26 Fe	6,403.84	6,390.84	7,057.98	705.0	705.0	718.5		
27 Co	6,930.32	6,915.30	7,649.43	776.2	776.2	791.4		
28 Ni	7,478.15	7,460.89	8,264.66	851.5	851.5	868.8		
29 Cu	8,047.78	8,027.83	8,905.29	929.7	929.7	949.8		
30 Zn	8,638.86	8,615.78	9,572.0	1,011.7	1,011.7	1,034.7		

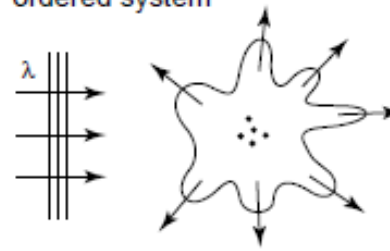


Scattering, Diffraction, and Refraction

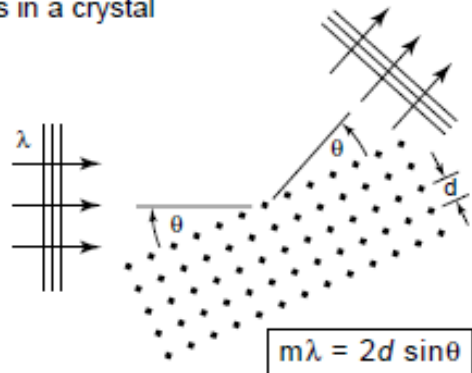
(a) Isotropic scattering from a point object



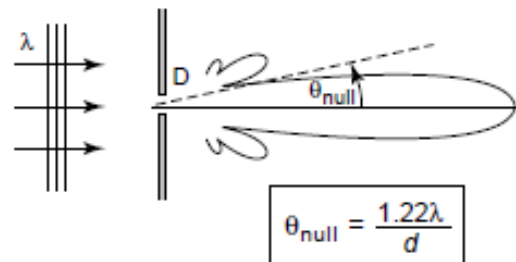
(b) Non-isotropic scattering from a partially ordered system



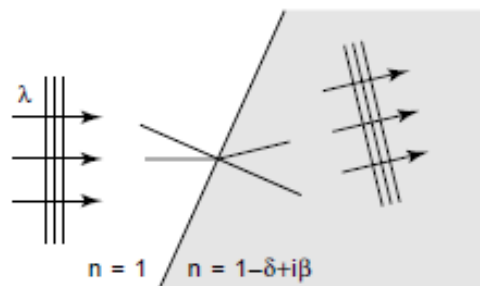
(c) Diffraction by an ordered array of atoms, as in a crystal



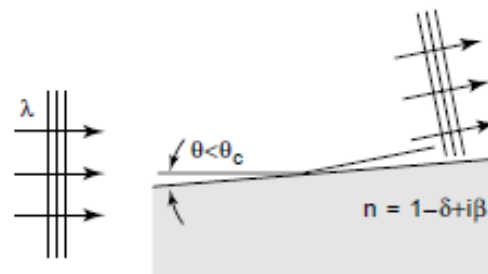
(d) Diffraction from a well-defined geometric structure, such as a pinhole



(e) Refraction at an interface



(f) Total external reflection





Periodic Table of the Elements



Group
IA

<div style="text-align: center;"> <p>Key</p> <p>Atomic number → 14</p> <p>Atomic weight → 28.09</p> <p>Density (g/cm³) → 2.33</p> <p>Concentration (10²² atoms/cm³) → 4.99</p> <p>Nearest neighbor (Å) → 3.57</p> <p>Name → Silicon</p> <p>Oxidation states (Bold most stable) → 4</p> <p>Symbol → Si</p> <p>Electron configuration → [Ne]3s²3p²</p> <p>Legend: Solid, Gas, Liquid, Synthetically prepared</p> </div> <p>References: International Tables for X-ray Crystallography (Reidel, London, 1983) (Ref. 44) and J.R. De Laeter and K.G. Heumann (Ref. 46, 1991).</p>																																																																																																																																																																																		
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Lanthanide series

58 140.12 0.7 2.91 3.85 Ce Cerium	59 140.91 0.4 3.04 3.93 Pr Praseodymium	60 144.24 0.5 3.05 3.93 Nd Neodymium	61 145 Pm Promethium	62 150.36 0.2 3.53 3.97 Sm Samarium	63 152.0 0.2 3.05 3.97 Eu Europium	64 157.25 0.5 3.50 3.90 Gd Gadolinium	65 158.93 0.4 3.53 3.91 Tb Terbium	66 162.50 0.5 3.51 3.94 Dy Dysprosium	67 164.93 0.8 3.27 3.49 Ho Holmium	68 167.26 0.4 3.47 3.48 Er Erbium	69 168.93 0.2 3.45 3.48 Tm Thulium	70 173.04 0.2 3.00 3.40 Yb Ytterbium	71 174.97 0.4 3.30 3.40 Lu Lutetium
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Actinide series

90 232.04 11.7 3.04 3.80 Th Thorium	91 231.04 15.4 4.01 3.21 Pa Protactinium	92 238.03 19.1 4.82 3.75 U Uranium	93 238 20.5 5.20 3.67 Np Neptunium	94 244 25.4 4.80 3.67 Pu Plutonium	95 243 11.9 2.94 3.67 Am Americium	96 247 10.8 2.90 3.67 Cm Curium	97 247 10.8 2.90 3.67 Bk Berkelium	98 251 10.8 2.90 3.67 Cf Californium	99 251 10.8 2.90 3.67 Es Einsteinium	100 257 10.8 2.90 3.67 Fm Fermium	101 259 10.8 2.90 3.67 Md Mendelevium	102 259 10.8 2.90 3.67 No Nobelium	103 262 10.8 2.90 3.67 Lr Lawrencium
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